A Standby server bulk arrival Queuing model of Compulsory server Vacation

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Abstract - We explore the steady state conduct of a M^θ/G/1 queue with compulsory vacation. Here the arrival follows a poisson distribution. Service is rendered in two stages in which the second stage is optional. After the completion of service, the server has to undergo a compulsory vacation. During the time of vacation for the continuous service process, a very important concept of standby server is introduced. This provides a complete satisfaction towards the customers. We obtain in closed form, the steady state probability generating functions for the number of customers in the queue for various states of the server, the average number of customers as well as their average waiting time in the queue and the system.

Key words: optional second stage, compulsory vacation, standby server.

1. Introduction

2. The mathematical description of the model
The model is based on the following assumptions:
a) Customers’ arrive one by one follows a compound Poisson process with a rate of arrival λ. Let λd_i dt(i = 1,2) be the first order probability that customers in batches of size i arrive at the system at a short interval of time (x, x+dt) , where 0≤ d_i ≤ 1. The concept of stand-by server starts to serve the customers when the original server is on vacation. We assume that the stand-by service time distribution follows an exponential distribution with stand-by service rate χ > 0 and mean stand-by service time 1/χ .
b) The server provides service in two stages with optional second stage of service. As soon as the first stage of the server closes; the server has the choice of taking a optional second stage of service with the likeliness γ. The service follows general
distribution. The service discipline is assumed to be on a first come first served basis (FCFS). Let us assume that the service time \( \delta_i(t, i = 1, 2) \) of the \( i \)th stage of service follows a general probability distribution with a distribution function \( M_i(x) \) and probability density function \( m_i(x) \). Let \( \delta_i(x) = \frac{m_i(x)}{1 - M_i(x)} \), \( i = 1, 2 \)

\( m_i(u) = \delta_i(u) e^{-\int_0^u \delta_i(x) dx} \), where \( i = 1, 2 \). 

(a) Once the service of a unit is finished, the server is accepted to take a compulsory vacation of general distribution. As soon as the first stage of the server closes and if no customer waits for optional second stage of service, then the server has the choice of taking a compulsory vacation with the likelihood \( \beta \). Otherwise, after the completion of customers second stage of optional service, vacation will be followed. After the culmination of necessary vacation, it joins the system to proceed with a service to the holding up clients. Give us a chance to expect the Compulsory vacation time to be an irregular variable, after the general conditional probability law with distribution function \( U(x) \) and density function \( u(x) \). Here, let us consider that \( \tau(x) \) is the conditional probability of a vacation period amid the \((x, x + dx)\), given that the slipped by time is \( x \), which can be given as

\[ \tau(x) = \frac{u(x)}{1 - u(x)} \]  

\[ u(t) = \tau(t) e^{-\int_0^t \tau(x) dx} \]  

(b)  

2.1. Notations

\( Q^{(i)}(t, x) \): Probability that at time \( t \), the server is active providing service and there are \( n(n \geq 0) \) customers in the queue excluding the one being served in the \( i \)th stage of service and the elapsed service time for this customer is \( x \). Consequently, \( Q^{(i)}(t) = \int_0^\infty Q^{(i)}(t, x) dx \) denotes the probability that at time \( t \) there are \( n \) customers in the queue excluding the one customer in the \( i \)th optional stage of service irrespective of the value of \( x \). \( S_n(x, t) \): Probability that at time \( t \), the server is on compulsory vacation with elapsed service time \( x \) and there are \( n(n > 0) \) customers waiting in the queue for service. Consequently, \( S_n(t) = \int_0^\infty S_n(x, t) dx \) denotes the probability that at time \( t \) there are \( n \) customers in the queue and the server is on compulsory vacation irrespective of the value of \( x \). \( R(t) \): Probability that at \( t \) time, there are no customers in the system and the server is idle but available in the system.

3. Steady state equation governing the system

\[ \frac{d}{dx} Q^{(1)}(x, t) + (\lambda + \delta_1(x)) Q^{(1)}(x) = \lambda \sum_{j=1}^{n-1} a_j Q^{(1)}_{n-j}(x) \quad n \geq 1 \]  

(3.1)

\[ \frac{d}{dx} Q^{(2)}(x, t) + (\lambda + \delta_2(x)) Q^{(2)}(x) = 0 \]  

(3.2)

\[ \frac{d}{dx} Q^{(1)}(x, t) + (\lambda + \delta_2(x)) Q^{(2)}(x) = \lambda \sum_{j=1}^{n-1} a_j Q^{(2)}_{n-j}(x) \quad n \geq 1 \]  

(3.3)

\[ \frac{d}{dx} S_n(x) + (\lambda + \tau(x) + \xi) S_n(x) = \lambda \sum_{j=1}^{n-1} a_j S_{n-j}(x) + \xi S_{n+1}(x) \]  

(3.5)

\[ \lambda R = \int_0^\infty S_0(x) \tau(x) dx \]  

(3.7)

Equations 3.1 – 3.7 are to be solved subject to the following boundary conditions:

\[ Q^{(1)}(0) = \int_0^\infty S_{n+1}(x) \tau(x) dx + \lambda a_{n+1} R \quad n \geq 0 \]  

\[ Q^{(2)}(0) = 0 \]  

\[ S_n(0) = \beta \int_0^\infty Q^{(1)}(x) \delta_1(x) dx + \int_0^\infty Q^{(2)}(x) \delta_2(x) dx \]  

(3.8)

3.1 Queue size Distribution at Random Epoch

We define the probability generation function as follows:

\[ Q^{(i)}(x, z) = \sum_{n=0}^\infty z^n Q^{(i)}_n(x), \quad Q^{(i)}(z, t) = \sum_{n=0}^\infty z^n S_n(x, t) \quad i = 1, 2 \]

\[ S_n(x, z) = \sum_{n=0}^\infty z^n S_n \]

\[ a(z) = \sum_{n=1}^\infty a_i z^i \quad |z| \leq 1 \]

4. Steady state queue size distribution at a random epoch

Cox (1955) has investigated non-Markovian models by changing them into Markovian ones, through the presentation of at least one supplementary factors. A stable recursive plan for the estimation of the restricting probabilities can be created, in view of an adaptable regenerative approach.

Multiplying eq. 3.1 by \( z^n \), summing over \( n \) and adding the result to eq. (3.2) and again using (A), we get

\[ \frac{d}{dx} Q^{(i)}(x, z) + (\lambda + \delta_1(x) - \lambda a(z)) Q^{(i)}(x) Q^{(i)}(x, z) = 0 \]

(4.1)
Similarly,
\[ \frac{d}{dx} Q^{(2)}_q(x, z) + (\lambda + \delta_2(x) - \lambda a(z)) Q^{(2)}_q(x, z) = 0 \]  
(4.2)
\[ \frac{d}{dx} S_q(x, z) + (\lambda + \tau(x) + \xi - \frac{\xi}{\bar{M}_2} - \lambda a(z)) S_q(x, z) = 0 \]  
(4.3)
Next, similar operations are carried out on the boundary conditions 3.8, we get

Applying (3.29) in (3.28), we get

Using the normalizing condition \( R + \int_0^\infty Q_q(x, z) \delta_1(x) dx = 1 \)

To find

Next we integrate 4.7

Similarly, \( Q^{(2)}_q(0, z) \) and \( S_q(0, z) \) are given in equations (4.4) - (4.6)

Next we integrate 4.7 - 4.9 with respect to x, by parts, we get

Where \( \omega = (\lambda + \xi - \frac{\xi}{\bar{M}_2} - \lambda a(z)) \)

\[ Q^{(2)}_q(0, z) = e^{-\lambda a(z) \frac{e^{-\lambda a(z) x}}{1 - M_1(\lambda - \lambda a(z))}} \]
(4.10)
\[ S_q(0, z) = \frac{1}{1 - U(\omega)} \]
(4.12)

Where \( M_1(\lambda - \lambda a(z)) = \int_0^\infty e^{-(\lambda a(z) - \lambda) x} \text{d}M_1(x) \) i.e. 1.2 is the Laplace Stieljes transform of the i-th service time and \( U(\omega) = \int_0^\infty e^{-\lambda a(z) x} \text{d}U(x) \) is the Laplace Stieljes transform of the compulsory vacation.

To find \( Q^{(2)}_q(x, z) \) and \( S_q(x, z) \) by applying the equations 4.10 - 4.12, we get

Using equations 4.13 -4.15 into equations 4.4 - 4.6 and further applying the equations 4.10 – 4.12, we get

Let \( D_q(x) = Q^{(1)}_q(x) + Q^{(2)}_q(x) + S_q(x) \) be the probability generating function of the queue size.

To determine the idle time \( R \)

Using the normalizing condition \( R + D_q(1) = 1, \) we get \( R \).

For this purpose we apply the following steps:

Eq.3.27 is indeterminate of the form \( \frac{0}{0} \) at \( z = 1 \). Hence L Hopital’s rule is applied. As a result we get

where \( E(I) \) is the mean size of the bulk arrival, \( E(M_1) \) is the mean service time, \( i = 1 \) and \( E(U) \) is the mean vacation time.

Applying (3.29) in (3.28), we get

Also we obtain the utilization factor \( \rho \) using the relation \( \rho = 1 - R \)
The imperative idea of standby server is to provide complete service to all the arriving customers. The ultimate goal is to have second optional service, random breakdowns, delay vacations and no renege customers. As a pending work, additional components of Priority organization have been displayed in this model since it sets up the authentic conditions for a comparable examination of execution strategies with this model can be figured. Additional piece of retrial line can moreover be considered.

5. Steady state mean queue size at a random epoch

Let $L_q$ denote the steady state mean queue size at a random epoch.

Since $D_q(z)$ is indeterminate of the form $\frac{0}{0}$ at $z=1$, we apply the following formula:

$$L_q = \lim_{z \to 1} \frac{D''N''' - N''D'''}{2(D'')^2}$$

(5.1)

Where double and triple primes denote the second and third order derivatives as follows:

$$D'' = (-\lambda E(I(l-1) - \xi)(1 - (\beta + \gamma)) + 2(-\lambda E(I) + \delta)^2 (1 - (\beta + \gamma))$$

$$N''' = (-\lambda E(I + \xi) (\lambda E(I)) (E(M_2) + 2\lambda E(M_2))$$

$$D''' = -3(-\lambda E(I) + \delta)^2 (\beta E(I)(E(M_1) + \lambda E(I)E(M_2) + E(M_2)) - 2(-\lambda E(I(l-1) - 2\xi)(-\lambda E(I) + \delta)(\beta + \gamma) +$$

$$+(-\lambda E(I) + 1 - \xi) (-\lambda E(I) + \delta)(\beta + \gamma))$$

(5.2)

Substituting (5.2) in (5.1), we obtain $L_q$ in a closed form. Further using $L_q$ in Little’s formula, we obtain the other required performance measures as follows:

Average number of customers in the system $L = L_q + \rho$ where $\rho$ is given by eq.4.22

Average waiting time in the queue $W_q = \frac{L_q}{\lambda}$

Average waiting time in the system $W = \frac{L}{\lambda}$

6. Conclusion

The perspective of discretionary stage of administration has been displayed in this model since it sets up the authentic conditions about the peculiarity of this work is the introduction of important concept, optional stage of administration in the knowledge which gives complete satisfaction to all the arriving customers. The imperative idea of standby server has been plainly all around characterized in this model which assumes an exceptionally conspicuous part in the greater part of the lining framework categories. This display has dormant helpful solid life application in media transmission structure, therapeutic interpretation and collecting organizations. As a pending work, additional components of Priority organization set up time and close down time can be fused and a comparable examination of execution strategies with this model can be figured. Additional piece of retrial line can moreover be considered.

References


