

# Improving MIMO Performance Using Water-Filling Algorithm

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**Abstract** - MIMO technology improves the performance of a wireless transmitting system at a time this technology able to send transmit and receive more than one signal, if we apply water-filling algorithm then the performance of MIMO technology is improved with respect to capacity, outage probability and SNR.

**Index Terms** - MIMO, Water-Filling Algorithm

## I. INTRODUCTION

In radio, multiple-input and multiple-output, or MIMO, is the use of multiple antennas at both the transmitter and receiver to improve communication performance. It is one of several forms of smart antenna technology. Note that the terms input and output refer to the radio channel carrying the signal, not to the devices having antennas.

MIMO technology has attracted attention in wireless communications, because it offers significant increases in data throughput and link range without additional bandwidth or increased transmit power. It achieves this goal by spreading the same total transmit power over the antennas to achieve an array gain that improves the spectral efficiency (more bits per second per hertz of bandwidth) or to achieve a diversity gain that improves the link reliability (reduced fading). Because of these properties, MIMO is an important part of modern wireless communication standards such as IEEE 802.11n (Wi-Fi), 4G, 3GPP Long Term Evolution, WiMAX and HSPA+. MIMO can be sub-divided into three main categories, precoding, spatial multiplexing or SM, and diversity coding.

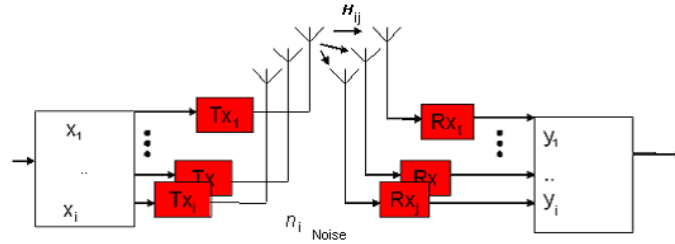
Precoding is multi-stream beamforming, in the narrowest definition. In more general terms, it is considered to be all spatial processing that occurs at the transmitter. In (single-layer) beamforming, the same signal is emitted from each of the transmit antennas with appropriate phase (and sometimes gain) weighting such that the signal power is maximized at the receiver input. The benefits of beamforming are to increase the received signal gain, by making signals emitted from different antennas add up constructively, and to reduce the multipath fading effect. In the absence of scattering, beamforming results in a well-defined directional pattern, but in typical cellular conventional beams are not a good analogy. When the receiver has multiple antennas, the transmit beamforming cannot simultaneously maximize the signal level at all of the receive antennas, and precoding with multiple streams is used. Note that precoding requires knowledge of channel state information (CSI) at the transmitter.

Spatial multiplexing requires MIMO antenna configuration. In spatial multiplexing, a high rate signal is split into multiple lower rate streams and each stream is transmitted from a different transmit antenna in the same frequency channel. If these signals arrive at the receiver antenna array with sufficiently different spatial signatures, the receiver can separate these streams into (almost) parallel channels. Spatial multiplexing is a very powerful technique for increasing channel capacity at higher signal-to-noise ratios (SNR). The maximum number of spatial streams is limited by the lesser of the number of antennas at the transmitter or receiver. Spatial multiplexing can be used with or without transmit channel knowledge. Spatial multiplexing can also be used for simultaneous transmission to multiple receivers, known as space-division multiple access. The scheduling of receivers with different spatial signatures allows good separability.

Diversity Coding techniques are used when there is no channel knowledge at the transmitter. In diversity methods, a single stream (unlike multiple streams in spatial multiplexing) is transmitted, but the signal is coded using techniques called space-time coding. The signal is emitted from each of the transmit antennas with full or near orthogonal coding. Diversity coding exploits the independent fading in the multiple antenna links to enhance signal diversity. Because there is no channel knowledge, there is no beamforming or array gain from diversity coding.

Spatial multiplexing can also be combined with precoding when the channel is known at the transmitter or combined with diversity coding when decoding reliability is in trade-off.

**II. MIMO CAPACITY**

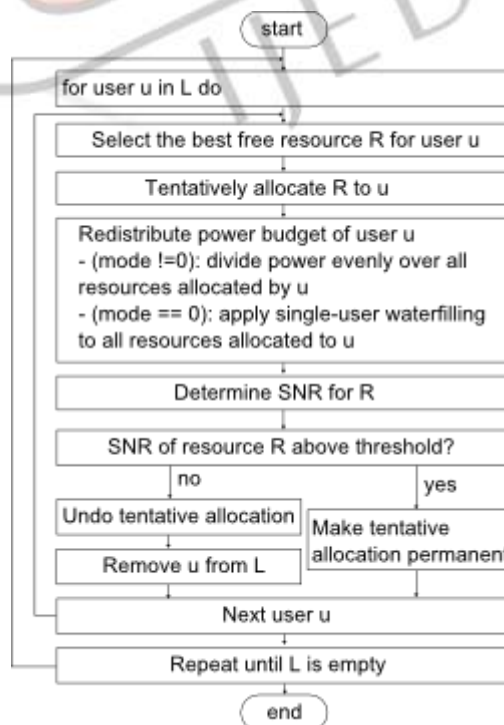


It is common to represent the input/output relations of a narrowband, single-user MIMO link by the complex baseband vector notation  $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$ , where  $\mathbf{x}$  is the  $(nT \times 1)$  transmit vector,  $\mathbf{y}$  is the  $(nR \times 1)$  receive vector,  $\mathbf{H}$  is the  $(nR \times nT)$  channel matrix, and  $\mathbf{n}$  is the  $(nR \times 1)$  additive white Gaussian noise (AWGN) vector at a given instant in time. Throughout the paper, it is assumed that the channel matrix is random and that the receiver has perfect channel knowledge. It is also assumed that the channel is memoryless, i.e., for each use of the channel an independent realization of  $\mathbf{H}$  is drawn. This means that the capacity can be computed as the maximum of the mutual information as defined in . The results are also valid when  $\mathbf{H}$  is generated by an ergodic process because as long as the receiver observes the  $\mathbf{H}$  process, only the first order statistics are needed to determine the channel capacity . A general entry of the channel matrix is denoted by  $\{h_{ij}\}$ . This represents the complex gain of the channel between the  $j$ th transmitter and the  $i$ th receiver. With a MIMO system consisting of  $nT$  transmit antennas and  $nR$  receive antennas, the channel matrix is written as:

$$H = \begin{bmatrix} h_{11} & \dots & h_{1nT} \\ \vdots & \ddots & \vdots \\ h_{nR1} & \dots & h_{nRnT} \end{bmatrix}$$

MIMO system with arrays of  $t$  transmit and  $r$  receive antenna. The transmitted signals in each symbol period are denoted by a  $t \times 1$  complex vector  $\mathbf{X}$ , where the  $x_i$  refers to the transmitted signal from antenna  $i$ . The total power of the complex transmitted signal  $\mathbf{X}$  is constrained to  $P$  regardless of the number of transmitted antennas. So  $\epsilon[\mathbf{x}^H \mathbf{x}] = \text{tr}(\epsilon[\mathbf{x}^H \mathbf{x}]) = P$ . The channel matrix  $H$  is deterministic in nature. Using singular value decomposition theorem,  $H$  can be denoted as:  $H = UDV^H$  Where  $U$  and  $V$  are  $r \times r$  and  $t \times t$  unitary matrices respectively. The columns of  $U$  are the eigenvector of  $HH^H$  in diagonal  $D_{ii}, i=1,2,\dots,r$  are called the singular value of  $H$  and denoted by  $\sqrt{\lambda_i}$  where  $i=1,2,\dots,r$ . By substituting the above value into eqn (5), the result will be  $\mathbf{y} = UDV^H \mathbf{x} + \mathbf{n}$  the overall channel capacity is the sum of sub channel capacity, which is given by:  $C = \sum_{i=1}^{r_0} \ln(1 + P_{ri}/\sigma^2)$  nats/s/Hz The power allocated to sub channel  $i$  is given by:  $P_{ri} = \lambda_i P/t$  Thus the channel capacity can be written as:  $C = \sum_{i=1}^{r_0} \ln(1 + P_{ri}/\sigma^2) = \ln \prod_{i=1}^{r_0} (1 + \lambda_i P/\sigma^2)$

**III. WATER FILLING ALGORITHM**



Flow Chart of Water-Filling Algorithm

- 1>Initialize  $p^0 = 0$ . [10][11]
- 2>At iteration n compute for  $k=1,2,\dots,K$  and  $j \geq k$  the coefficients  $\alpha_{k,j}^{(n)} = h_k^H \sum_{k,j}^{-1} (p^{(n-1)}) h_k$ .
- 3>Water-filling step: let  $\gamma^{(n)} = \arg \gamma_{\geq 0} \sum_k^{max} \gamma_{k \leq p} \sum_{j=1}^K \Delta_j \sum_{k=1}^j \log(1 + \gamma_k \alpha_{k,j}^{(n)})$
- 4>update step: let  $P^{(n)} = (1/K) \gamma^{(n)} + (1 - (1/K)) P^{(n-1)}$

By applying water-filling algorithm the performance of a MIMO system is improved. When a communication channel is corrupted by severe fading or by strong intersymbol interference, the adaptation of transmit signal to the channel condition can typically bring a large improvement to the transmission rate. Adaptation is possible when the channel state is available to the transmitter, usually by a channel estimation scheme and a reliable feedback mechanism. With perfect channel information, the problem of finding the optimal adaptation strategy has been much studied in the past. If the channel can be partitioned into parallel independent subchannels by assuming i.i.d. fading statistics for the fading channel, or by the discrete Fourier transform for the intersymbol interference channel, the optimal transmit power adaptation is the well-known water-filling procedure.

In water-filling, more power is allocated to "better" subchannels with higher signal-to-noise ratio (SNR), so as to maximize the sum of data rates in all sub channels, where in each sub channel the data rate is related to the power allocation by Shannon's Gaussian capacity formula:  $1/2 \log(1 + \text{SNR})$ . However, because the capacity is a logarithmic function of power, the data rate is usually insensitive to the exact power allocation, except when the signal-to-noise ratio is low. This motivates the search for simpler power allocation schemes that can perform close to the optimal.

IV. ADAPTIVE TRANSMIT POWER ALLOCATION

When channel state information is known at the transmitter the capacity can be increased by allocating transmit power to different antennas using Water filling algorithm [4]. [7]

$$P_i = (\mu - \sigma^2 / \lambda_i)^+$$

Where  $i=1,2,3,\dots,r_0$  ... and  $\mu$  is chosen so that  $\sum_{i=1}^{r_0} P_i = P$

and  $a^+$  denotes  $\max(a, 0)$ . So the received signal power at  $i$ th sub channel is given by:  $P_n = (\lambda_i \mu - \sigma^2)^+$

Thus the channel capacity can be written as:

$$C = \sum_{i=1}^{r_0} \ln(1 + (\lambda_i \mu - \sigma^2) / \sigma^2) = \sum_{i=1}^{r_0} \ln(1 + (\lambda_i \mu / \sigma^2)^+) = \sum_{i=1}^{r_0} \ln(\lambda_i \mu / \sigma^2)^+ \text{ nats/Hz}$$

V. RESULTS

A. Figures and Tables

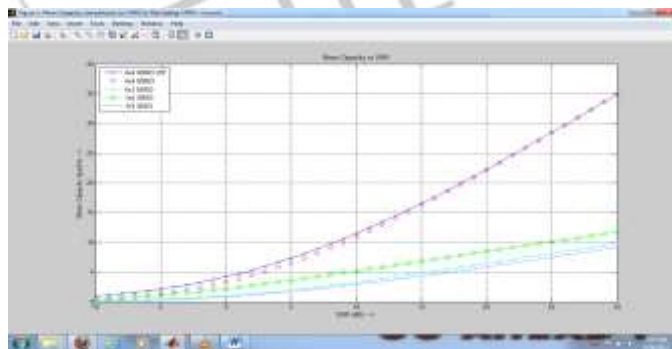


Fig1: capacity vs snr curve (improve in water filling algorithm)

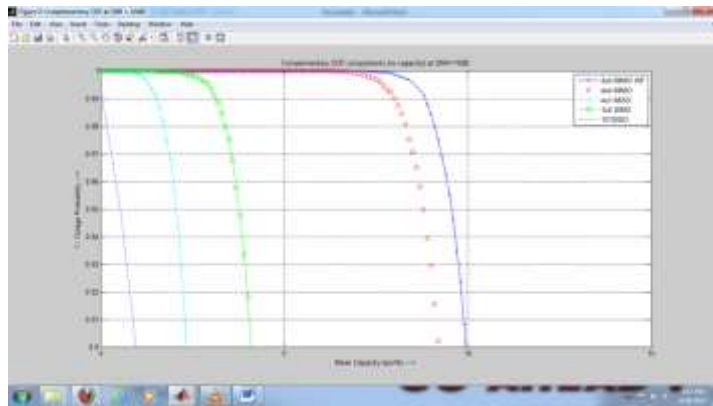


Fig2: outage probability vs capacity(improve in water filling algorithm)

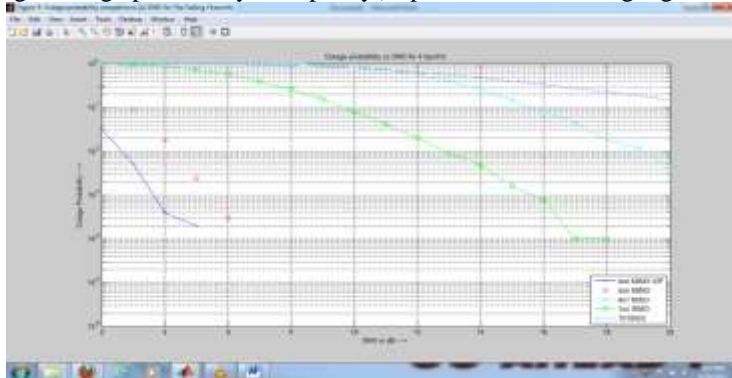


Fig3: outage probability vs snr(improve in water filling algorithm)

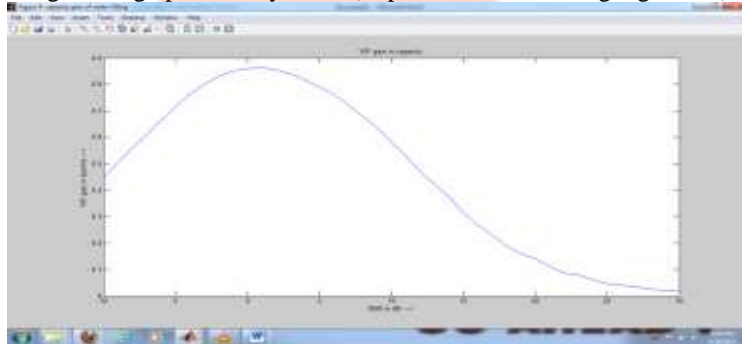


Fig4: water filling algorithm implementation

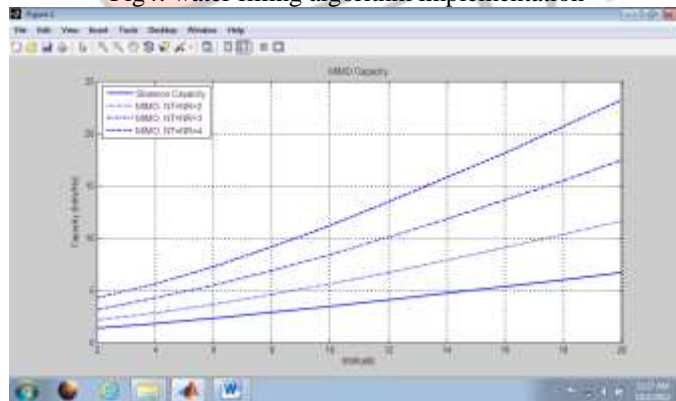


Fig5: Capacity of ordinary Shannon Channel

## VI. CONCLUSION

MIMO inherently possess spatial diversity, which increases robustness of the system by eliminating fades. Using MIMO the effective SNR of the system, thereby system throughput can be increased with the aid of spatial multiplexing. But in other hand it impose challenge for designing such a cost effective end user MIMO system. Approximate power adaptation algorithms are investigated in this paper. A rigorous performance low bound for sub-optimal power allocation is derived. A very low complexity constant-power adaptation method is proposed using the bound derived. By applying water filling algorithm the MIMO performance is improved.

**VII. REFERENCES**

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