Comparative FE Analysis of Pressure Vessel of Hemispherical, Ellipsoidal and Torospherical End Connection

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Abstract - this paper presents analysis results of stresses in pressure vessel of hemispherical, ellipsoidal and torospherical end connection. Here Von-Mises stresses membrane stresses, total displacements and total mass of model are obtained for connections. The connections are the weakest areas where high stresses are seen. The finite element analysis software is used to analyze Shell-End connections. These connections are highly stressed joints. The results are obtained from 3-D Finite Element Analysis (FEA).

Index Terms - Hemispherical end connection; ellipsoidal end connection; torospherical end connection; stresses; finite element analysis, pressure vessels

I. INTRODUCTION

A container designed to hold gases or liquids which works, at a pressure substantially different from the ambient pressure and supposed to be leak-proof is known as Pressure vessel. They may have different configurations of shapes and sizes, but basically recommended shapes are made of sections of spheres, cylinders, and cones. A common design is a cylinder with hemispherical end caps connection.

A spherical shape is difficult to manufacture, therefore more expensive, so most pressure vessels are cylindrical with 2:1 semi-elliptical end connections or end caps on each end [2].

Design of pressure vessel is considered on the basis of the most likely mode of damage or failure, the method of stress analysis employed, significance of the results, the selection of material type and its linear-nonlinear behavior as a result of the external forces on the vessel. The Theories of failures considered by the bodies like American Society of Mechanical Engineers (ASME standard, Section VIII, Div-2, 2007) for pressure vessels are considered here. Pressure vessels are characterized by the existence of stresses along three axes. First of all, due to pressure, there is a principal stress directed as the pressure itself and thus orthogonal to the wall of the vessel. Two additional principal stresses act on the plane orthogonal to the previous one. In the case of cylindrical elements the first of such stresses is radial; the other two are directed, respectively, along the circumference and along the axis of the cylinder. The most generally accepted failure theories for ductile materials, such as steel used to build pressure vessels, are the well-known theory of maximum shear stress of Guest–Tresca, and the other one is distortion energy theory of Huber–Hencky [5].

The simplified thin shell methods, illustrated by application to a pressure vessel that has many of the geometric and operational features of a pressurized vessel were used. A study of major sensitive areas was performed for a pressure vessel in this research work on pressure vessels [3]. A cylindrical pressure vessel with hemispherical, torospherical and ellipsoidal end connections, involving the partial differential equations for the classical theory of thin shells of revolution axisymmetric in character were attempted, using a step-by-step integration procedure and a segmentation technique. The numerical results were obtained with generalized computer program developed in FORTRAN IV and then compared the results with ASME code [4]. In pressure vessels the center lines of the cylinder and dome portions often do not coincide at their junction. To predict accurately the stress and displacement distributions around the discontinuity, an iterative finite element scheme was developed using a conical shell finite element. The method was applied to three types of pressure vessels, one with hemispherical end domes, second with ellipsoidal end domes and third with torospherical end domes. The method yielded satisfactory results and showed good convergence [6].

A numerical analysis using the finite element method has been carried out to investigate the design sensitivity of thin torospherical end pressure vessels. The sensitivity of the end stresses to particular forms of shape imperfections were analyzed numerically and compared with those in the corresponding ‘perfect’ end which were presented [7].

II. FE ANALYSIS OF PRESSURE VESSEL WITH DIFFERENT TYPES OF END CONNECTION

The connections are the weakest areas where high stresses are seen. There are some conventional formula’s which are used for the calculation of stresses in these areas. ASME section VIII div 2 has realized the power of Finite element method and has used it to analyze all the critical areas where weld and bolted connections are plotted and are difficult to analyze via conventional formulas. Finite element method is used to solve these types of problems. Pro-Mechanica software was used in this research work to analyze shell connection to shell-end connections. These connections are highly stressed joints[1].
Table 1 Specification of pressure vessel

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner diameter (D)</td>
<td>5000</td>
<td>Mm</td>
</tr>
<tr>
<td>End connection thickness (t)</td>
<td>25</td>
<td>Mm</td>
</tr>
<tr>
<td>Shell thickness (T)</td>
<td>50</td>
<td>Mm</td>
</tr>
<tr>
<td>Internal Pressure (P)</td>
<td>2</td>
<td>MPa</td>
</tr>
<tr>
<td>Global Temperature (G.T.)</td>
<td>120</td>
<td>°C</td>
</tr>
<tr>
<td>Reference Temperature (R.T.)</td>
<td>30</td>
<td>°C</td>
</tr>
</tbody>
</table>

### III. SHELL WITH HEMISPHERICAL END CONNECTION

Diameter and thickness are the governing parameters for geometry of hemispherical end. The most important part which is considered in FE modeling here, is the consideration of joining the hemispherical end of thickness 25 mm with inner diameter as 5000 mm to a shell with inner diameter of 5000 mm but of thickness 50 mm. The welding strength is modeled in a typical way as explained here under.

The number of curves resulting to surface is modeled and the increasing thickness shell property is assigned to these separate shell element from 25 mm of hemispherical end to 50 mm of the shell. To make it more clearly here the hemispherical end with 3 curves with increasing thickness from 25, 35, and 45 which is then connected to the shell of 50 mm thickness has been modeled. The diagram below depicts the assumption of modeling clearly.

The mid surface shell model is modeled with the diameter of 5050 mm (considering the thickness of 50 mm of the shell) and the radius of mid-surface hemispherical end is 2512.5 considering the thickness of 25 mm. The joining of the two non-tangential arcs is done using small line segment tangential to the hemispherical end while is in touch with the shell. The line is defined with the property of shell with thickness of 40 mm encompassing the strength of the weld joining the two base metals [8].

![Fig.1 Hemispherical end connection](image)

![Fig.2 Sub-structured meshed model for hemispherical type pressure vessel](image)

![Fig.3 Results of Von-Mises stresses of hemispherical end connection](image)

Max Von-Mises stress found in connecting zone is around 110-130 MPa
IV. SHELL WITH ELLIPSOIDAL END CONNECTION

This case which is analyzed is when the hemispherical end changed to ellipsoidal end connection with the cylindrical vessel. The model was prepared with similar modeling practice of using three surfaces where one is allotted the thickness of 45 mm, the second which is of 35 mm thickness and the connection which is of 25 mm thickness.

The use ellipsoidal end results in the lengthwise reduction of the vessel. If the length of the hemispherical end is x, the length of ellipsoidal end will be x/2 mm resulting the reduction in the cost of the material [8].

Fig. 4 Results of membrane stresses of hemispherical end connection Max membrane stress found in connecting zone is around 90-110 MPa

Fig. 5 Results of total displacement of hemispherical end connection Total displacement is 1.21 mm

Fig. 6 Ellipsoidal end connection

Fig. 7 Sub-structured meshed model for ellipsoidal type vessel

Fig. 8 Results of Von-Mises stresses of ellipsoidal end connection Max Von-Mises stress found in connecting zone is around 220-250 MPa

Fig. 9 Results of membrane stresses of ellipsoidal end connection Max membrane stress found in connecting zone is around 180-200 MPa

Fig. 10 Results of total displacement of ellipsoidal end connection Total displacement is 7.68 mm
V. SHELL WITH TOROSPHERICAL END CONNECTION

Shell with Torospherical End Connection This case which is analyzed is when the hemispherical end is changed to torospherical end connection with the cylindrical vessel.

The use torospherical end results in the lengthwise reduction of the vessel. Here the length is reduced to nearly thrice the one which is with hemispherical end but at the same time the knuckle radius is used here for reduction of the end stresses. The meshed model is shown in the figure below which will be analyzed with similar boundary conditions as was done in hemispherical type of sub-structured model [9].

![Fig.11 Torospherical end connection](image1)

![Fig.12 Sub-structured meshed model for torospherical type pressure vessel](image2)

![Fig.13 Results of Von-Mises stresses of torospherical end connection](image3) Max Von-Mises stress found in connecting zone is around 900-1060 MPa

![Fig.14 Results of membrane stresses of torospherical end connection](image4) Max membrane stress found in connecting zone is around 450-545 MPa

![Fig.15 Results of total displacement of torospherical end connection](image5) Total displacement is 1.78 mm

VI. RESULTS

<table>
<thead>
<tr>
<th>Description</th>
<th>Hemispherical end vessel</th>
<th>Ellipsoidal end vessel</th>
<th>Torospherical end vessel</th>
<th>Stress predicted by ASME Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Von- Mises stresses (MPa)</td>
<td>110-130</td>
<td>220-250</td>
<td>900-1060</td>
<td>282</td>
</tr>
<tr>
<td>Membrane stresses (MPa)</td>
<td>90-110</td>
<td>180-200</td>
<td>450-545</td>
<td></td>
</tr>
<tr>
<td>Total displacement (mm)</td>
<td>1.21</td>
<td>7.68</td>
<td>1.78</td>
<td>-</td>
</tr>
<tr>
<td>Total mass (Tonne)</td>
<td>8.734</td>
<td>6.009</td>
<td>5.148</td>
<td>-</td>
</tr>
</tbody>
</table>
VII. CONCLUSION

- From the stress analysis, Von-Mises and membrane stresses of ellipsoidal endconnection are almost double than hemispherical end connection but, all are in the allowable range of stresses as depicted by ASME section VIII Div-2. So failure will not occur at hemispherical and ellipsoidal end connection.
- Stresses of torospherical end connection are almost four times higher than ellipsoidal end connections which are greater than an allowable range of stresses as depicted by ASME section VIII Div-2. So, failure will occur at torospherical end connection and thus advised not to be used same thickness.
- The weight of ellipsoidal end connection is 0.68 times that of hemispherical end connection which saves ≈2.725 tonne of material with the current size and modeling of pressure vessel.

Thus, ellipsoidal end connection is most favorable than hemispherical and torospherical end connection in industry.

ABBREVIATIONS

- D: Inner diameter of pressure vessel, mm
- H: Height of ellipsoidal end connection, mm
- P: Internal pressure, MPa
- t: Thickness of shell end connection, mm
- r: Knuckle radius, mm

APPENDIX A

Table 3 Failure theory of pressure vessel with material description

<table>
<thead>
<tr>
<th>Material</th>
<th>UTS at ambient temp. (MPa)</th>
<th>Yield stress at ambient temp. (MPa)</th>
<th>Primary allowable stress as per ASME (remains same at 214˚C) (MPa)</th>
<th>Design stress as per ASME (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>At ambient temp.</td>
</tr>
<tr>
<td>SA 516 Gr. 70</td>
<td>482</td>
<td>282</td>
<td>137.89</td>
<td>160</td>
</tr>
<tr>
<td>Inconel 600 SB-168</td>
<td>552</td>
<td>241</td>
<td>157.8</td>
<td>160.6</td>
</tr>
</tbody>
</table>

REFERENCES