

Steering of Multi-State Systems by Modeling and Simulation

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Abstract - In recent years, Multi-State Systems (MSS) have been rigorously studied because of their inherent complexity. In this paper, we present an approach for steering multi-state systems in an industrial context, over a given life cycle. This approach aims at identifying the best choices for maintenance policies of Multi-State Systems, according to transition parameters of states and performance indicators chosen. The degradation process of the system, based on the Markov model, has been transformed with a dynamic Bayesian model to utilize its advantages. The decision-making of maintenance policies through the Dynamic Bayesian Network is examined, as well as, compared to the simulation results obtained from different test cases.

Index Terms - Multi-State Systems; Markov Graph; Dynamic Bayesian Network; Modeling; Preventive Maintenance,

I. INTRODUCTION

Maintenance is at the height of the performance of any industrial activity in particular when the systems become more complex. So the maintenance departments must study, analyze, or develop strategies constantly, as in [1] in order to supervise production systems often subjected to some of the negative influences. These influences may be due to:

- design flaws;
- the manufacturing process;
- an industrial operating of production system (heavy loads of solicitation);
- the installation procedures;
- the aggressiveness of the working environment (dusts, temperature, vibrations, shocks, humidity, impurities,...);
- the variability of production policy (sudden increase in production capacity, less downtime,...).

Under the effect of influences as stated above, production systems can operate with different levels of performance hence the name of Multi State System (MSS). A system consists of several components and where each one plays a given function. The theory of binary systems for example in [2] admits only two possible states for the system itself or its components: operating perfect state or failed state. In fact, in the actual operation of production systems, there may be several intermediate states; each state corresponds to a specific performance level (Fig.1). This is mainly due to the vagaries of production, the degradation of components and some environmental impacts).

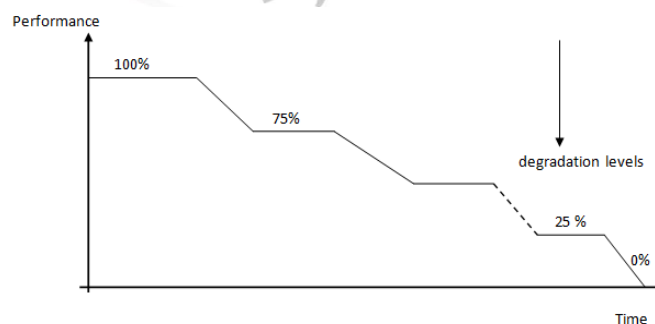


Fig. 1. Multi-state quantification of degradation

In practice, the components of a system can operate with different levels of damage. The degradation varies from operating states to total failure of the system, as reported in [3]. For example: a cotton gin can operate with the states 0, 1, 2, 3, 4 corresponding to 100%, 75%, 50%, 25%, 0% of its total capacity. One should notice that a state can either be ordered or continuous.

The performance of any system depends on the state of the components and there are different configurations such as: the systems in series, parallel, series-parallel and parallel-series, k-out of-n, consecutive k-out of-n. Consecutive k-out of-n systems are also the subject of interesting studies considering their better reliability compared with series systems, cheaper than parallel systems, [4].

Therefore it is more than necessary to model Multi-State Systems, to not only understand their behaviors in function of the performed maintenance actions, but to also choose a preventive maintenance policy with an optimal compromise of transition rates and cost.

The objective of this paper is to present, from a degradation model, an approach whose principal objective is to improve the maintenance policy for Multi-State Systems in an industrial context. In addition, we strive to maintain in operational conditions the system taking the production constraints into account (availability, cost).

Our work is based on the identification of the choice of an optimal maintenance policy among several possible configurations for the maintenance operations of Multi-State Systems through the simulation.

Some approaches exist to simulate the behaviors of the materials as displayed in [5], but it treats in line with the predicted reliability. With regard to the complexity of multi-state study, their reliability's calculation, a large number of work has used the methods of framing [1] in order to well address these issues. But another specificity of multi-state systems is in their modeling and characterization, as mentioned in [6].

However, we found in the literature markovian multi-state models, particularly in biostatistical domain ([7], [8]). A degradation model that allows to carefully describe the degraded states occupied by the system over time is said to be a markovian if the information on the previous states are summarized by the present states.

Recently, a considerable numbers of studies that deal with multi-state systems on the evaluation of performance proposed more reliable approaches, mostly ([6], [9]) without involving the maintenance cost). Some work in [10] refers to a preventive maintenance optimization by genetic algorithm with probabilistic graphical model MGD (Time Model Graph). Indeed, the authors of this work are not interested in the study of system availability.

In this research, we start from a markovian degradation model. This model represents the set of successive degradations states of a system with integrated maintenance process which is defined according to a given maintenance policy:

- Minimale préventive maintenance ;
- Corrective minimale maintenance.

From this model, we propose the "Bayesian multi-state" which is a very novice model that incorporates the performance indicators (maintenance cost, availability) for the maintenance policy evaluation. The choice of formalism Dynamic Bayesian Network (DBN) for our model is of double interest:

- 1) Dynamically get the system throughout its life cycle and decide via the decision node, which maintenance policy should be preferred;
- 2) Obtain a less dense states model may be occupied by the system which we want to model the behavior [11].

The identification of the choice of the maintenance policy is conducted via simulation according to different configurations of transition rates between the states occupied by the system over time. The model study is carried out at different angles depending on whether or not the transition rates between the different states of the system vary over time.

Our contribution is a part in a methodological level and modeling through our proposed approach and the other hand practical in the industrial context where multi-state models are rarely used.

The remainder of the paper is organized as follows. Section II is devoted to a related work on MSS. Section III gives the formalism of the Dynamic Bayesian Network (DBN) and the Markov process. Section IV suggests the proposed approach for steering. Section V is the formulation of the proposed approach. Section VI gives the results of the simulation. Section VII provides a conclusion of the work.

Due to its complexity and its imminent importance in the real world, studying and understanding MSS has recently been the subject of huge debates and the ground of countless arguments. Finding evidence to support that MSS has been very useful to scientists, engineers and researchers is not a rare luxury at all. We give then in the next section related work on MSS (Fig.2).

II. RELATED WORK

Multi-State Systems (MSS) have been of great interest for researchers. Some work in [12] generalized the theory of the binary coherent systems for multi-state components. In their projects, they considered the state of the system is the bad component state in the best way minimal or component state of the best component in the worst minimum cut. Reference [13] developed a basic theory for the study of components and systems that may have an infinite number of states. They asked axioms extending the standard notion of a coherent system of the new multi-state coherent systems and obtain deterministic and probabilistic properties. Work in [14] presented three types of coherence based on the strength of the relevant axioms and studies two of them.

In fact, these work on the mathematical theory of MSS provided several basic models, including the model of Barlow, the model of El-Newehi and Sethuraman, the model of Griffith ..., in [4].

Other authors have also strengthened the theory of Multi-State with more or less different approaches based on approximation methods or estimation and simulation ([15]).

Many studies have been done on the analysis of the availability or reliability of Multi-State Systems (MSS). However, we have identified four approaches in the literature to assess the availability and reliability of Multi-State Systems (MSS):

- Stochastic [16],
- Monte Carlo [17],
- Functional [18],
- UMGF [19].

As for estimating the availability of larger systems, the UMGF (Universal Moment Generating Function) method is the best applied among other methods (Stochastic, Monte Carlo). A literature review relatively exhaustive on the reliability of MSS can be accessed in [20].

Some work focused on the maintenance policy optimization of MSS:

Reference [21] was used the Markov approach on a system with three states. He conducted a study of the state of availability, failure frequency and average duration of failure.

Reference [22] gave an overview of the performance measures (availability, production rate,...) of monotone Multi-State Systems observed in the given meantime. He sets out the calculation of these measures and provides easy approximation formulas. The accuracy of these formulas is studied by making comparisons with the results obtained by Monte Carlo simulation.

Reference [23] developed a simulation algorithm, which can be used in place of a system structure function for calculating the probability distribution of the system state. He also used the theory of Markov chain to give the reliability of components and system. Two models are proposed to determine the probability of the system states distributed over several periods. The probabilities of transitions from one state to another are assumed to be known so they do not use the transition rates.

Reference [24] developed a model for assessing availability, production rate and reliability function of multi-state degraded subjected to minimal repairs and imperfect preventive maintenance. They associate to each state of system of their Markov model a rate performance. The objective is that the performance rate of the system at time t is greater than the customer's request.

Reference [25] proposed a study and construction of a general model for representing generic term models that can be adapted to multi-state systems, with the laws of any stay time and possibly a contextual dependency. To do this, they offered a particular Dynamic Bayesian Network named Model Graphical Duration (MGD).

Reference [26] established an integrated planning of preventive maintenance and production of multi-state systems, the work provides planning templates to generate an optimal production plan at the tactical level and the moments when intervention intervals for preventive maintenance actions (acyclic or cyclic). To obtain optimal solutions, they developed methods for evaluating the durations and costs of maintenance capabilities relating to systems and some algorithms of resolution. This work provides an economic impact by integrating the planning of preventive maintenance and production.

Reference [27] proposed an approach based on a dynamic Bayesian model named "multi-state". This work, studies a degradation model, provides the analysis and identification of better maintenance policy for multi-state systems.

Seen from that point of view application, the multi-state approach was extended in various fields such as industry, medicine etc., through the work in ([28], [29]). We give here a review categorization of the work on MSS.

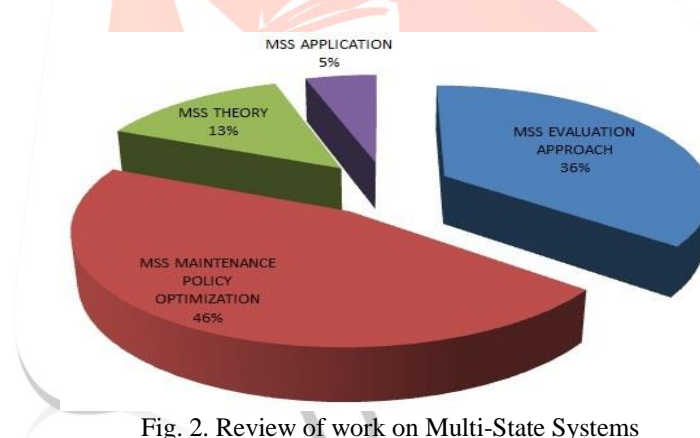


Fig. 2. Review of work on Multi-State Systems

III. MARKOV CHAIN FORMALISM AND DYNAMIC BAYESIAN NETWORKS

We present here Dynamic Bayesian Networks and Markov Chain while exposing the strengths and benefits of each.

Dynamic Bayesian Networks

Dynamic Bayesian Network (DBN) is really, an extension of Bayesian network in which the temporal evolution of the variables is represented ([30], [31], [32]). Dynamic Bayesian Network is also shown as an extension of Markov Chains [33].

It aims to model the probability distributions of a series of variables $(X_t)_{1 \leq t < T} = (X_{1,t}, X_{2,t}, \dots, X_{n,t})_{1 \leq t < T}$ on a sequence of length $T \in \mathbb{N}$. The process is represented by a node X_t^i at time step t with a finite number of possible states $S_{X_t^i}$. A state space Ω is the cross product of the values of states for individual state variables: $\Omega = \prod_{i=1}^N S_{X_t^i}$. $p(X_t^i)$ being the probability distributions on variable states at step time t . The nodes correspond to state variables that can be partitioned into two sets: one corresponding to the state variables at time step t and the other corresponding to the system state at next time step ($t+1$). The variable is then represented at successive times in this case.

The following figure shows a Dynamic Bayesian Network with two time steps k and $(k+1)$, the network is called Dynamic Bayesian Network with two slices named 2-DBN.

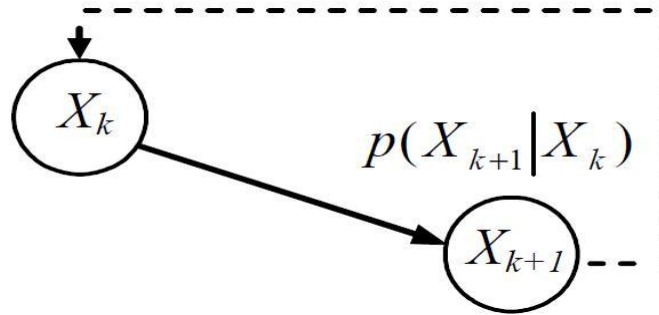


Fig. 3. Modeling of a 2-DBN

A Conditional Probability Table (CPT) of the node X_{k+1} depending on the node X_k in Fig.3, is given in the following table:

Table 1: Example of CPT

		X_{k+1}	
		Perfect	Failed
X_k	Perfect	$1-\lambda$	Λ
	Failed	M	$1-\mu$

The parameters λ and μ are respectively the failure rate and the repair rate. The node X_{k+1} tells us about the probability that the system be in the perfect state or failed state in time. At the moment these probabilities become those of the node X_k (this transition probability is represented by the dotted line in the Fig.3).

In many projects dedicated to the representation of complex systems, probabilistic graphical models such as DBN hold a prominent place in the modeling of dynamic systems with discrete and finite states [34].

Many studies stretch on the relationship or the link between the Markov chains and the Dynamic Bayesian Network. In these studies, a strong correspondence between the Markov chains and the Dynamic Bayesian Network is studied and presented on the case by case method. Furthermore, they displayed the undeniable advantage of Bayesian network on Markov chains [35].

Consequently, one can proudly state that the Dynamic Bayesian Network is the most suitable and appropriate in the study of the reliability analysis of large complex systems.

Markov chain

The sequence of random variables X_1, X_2, \dots, X_n forms a Markov chain with discrete states space if for all $n \in \mathbb{N}^*$ and all possible values of X_n random variables, we have :

$$i_1 < i_2 < \dots < i_n$$

$$P(X_n = j | X_{n-1} = i_{n-1}, X_{n-2} = i_{n-2}, \dots, X_1 = i_1) = P(X_n = j | X_{n-1} = i_{n-1}) \quad (1)$$

This conditional probability $P(X_n = j | X_{n-1} = i_{n-1})$ is called the transition probability.

Indeed the transition probability allows for a transition from E_i state at step (n-1) to the E_j state at step nth. The Markov chain is said to be homogeneous when this probability does not depend on n, that is to say

$$P_{ij} = P(X_n = j | X_{n-1} = i_{n-1}) \quad (2)$$

The following figure shows an example of a Markov chain with two states; the model represents the transition probabilities that are associated with each arc.

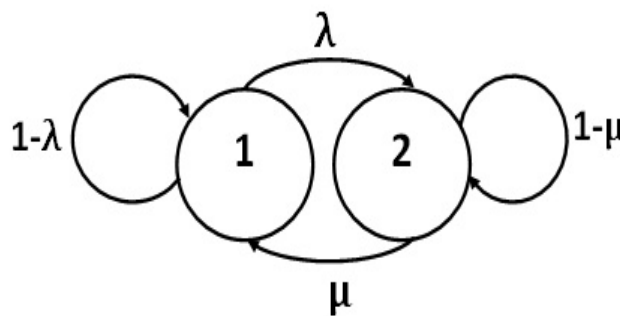


Fig. 4. Example of Markov Chain

In the case of a Markov process with time independency, the failure rate is considered to be constant while in the time dependent this failure rate of Markov process is not constant; hence variable. This is explained by the fact that degradation of the component of a system within an industrial environment is constantly changing over time due to its use or age.

However the use of Markov models faces some limitations in particular the combinatorial explosion in the number of states likely to be occupied by the system which is desired to model the behaviour.

IV. APPROACH

Our goal is to provide an approach and a tool for decision support that enable to search the optimal preventive maintenance policy based on the simulation of material on an entire operating horizon, with a learning gradually(history) decisions and performance (failure rate and availability) obtained each time. In addition, the tool should allow incorporating expert knowledge of a cognitive nature. As a dysfunctional representation of the equipment, we start from a state-space model via a graph of Markov chain:

1. we produce the structure of a Dynamic Bayesian Network (DBN) and define the Conditional Probability Tables (modeling of the transition parameters of Multi-State Systems). The nodes correspond to the transition parameters;
2. we define the rules for the passage of the Markov graph to a Dynamic Bayesian Network (setting rule of Conditional Probability Tables CPT, ...). These are generic rules of passage and not specific to application case treated;
3. we integrate the performance indicators for the assessment (availability, maintenance cost, ...) in the DBN;
4. we simulate the behavior of the equipment (multi-state) on a service life where the parameters are stochastic and not constant. The target node is the variable state of degradation. The stochastic evolution of preventive maintenance levels will be associated with each iteration, the availability and cost of operational maintenance of equipment;
5. we use a reinforcement learning algorithm to obtain the optimal level of preventive maintenance in view of the simulation.

V. APPROACH FORMULATION

In this study, we consider the degradation process of a production system, in our study. The lifecycle of the system is supported by a given maintenance policy.

In the next subsection, we will first describe and define the behavior of MSS in general.

General description of Multi-State System model

The study of any process must go through a modeling. Here we describe a general model of MSS with the following assumptions:

- the system can take several possible levels of degradation;
- the maintenance policy adopted for the system varies and can be a preventive maintenance (minimum, medium, maximum) or minimal corrective maintenance;
- the failures of system are statistically independent;
- the system is repairable.

The system start with the initial operating state said perfect to occupy over time degraded states or failed states completely (Fig.4).

Transition rates between the states are:

μ_{dn} : repair rate from degraded state to normal state

λ_{dn} : failure rate from normal state to degraded state

μ_{df} : repair rate from degraded state to failed state

λ_{df} : failure rate from degraded state to failed state.

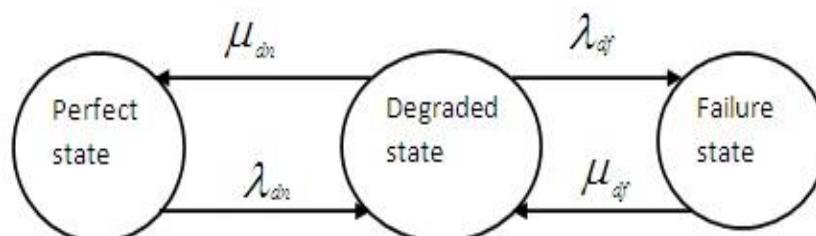


Fig. 5. General description of Multi State Systems

In the next subsection, we rely on a multi-state Markov model.

Multi-state Markov model

We consider a Multi-State System following the Markov. The system undergoes a maintenance process where the constraints are:

- Less downtime ;
- Master the degradations ;
- Identify ways to assess the maintenance policy adapted;
- Understand the behavior of the system over time.

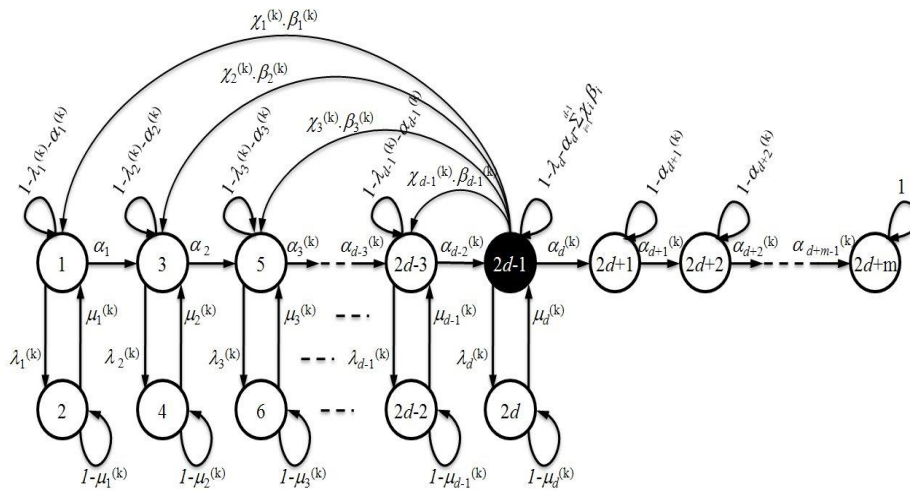


Fig. 6. Markov model graph [25]

states i vary from 1 to n , with $n=2d+m$

state1: perfect operating

degraded states $(2j-1): j= 2, \dots, d$

states $(2j):$ failed from operational state with $j=1, \dots, d$

states $(2d+k):$ failed after a degradation process with $k=1, \dots, m$.

The parameters of transitions between states are:

λ_j : failure rate from state $(2j-1)$ to state $(2j)$ with $j=1, \dots, d$

α_j : degradation rate from $(2j-1)$ state to $(2j+1)$ state with $j=1, \dots, d$

β_j : rate of passage from degraded state $(2d-1)$ to previous degraded state $(2j-1)$ with $j=1, \dots, d-1$.

μ_j : repair rate from failure state $(2j)$ to degraded state $(2j-1)$ with $j=1, \dots, d$.

To dynamic bayesian model: Transition rules

To develop our multi-state dynamic bayesian model (Fig. 7), we establish the rules of passage from the multi-state Markov model (Fig.6).

The transition rules are:

- Parameters transformation of transition rates in to nodes ;
- Transformation of the states occupied by the production system over time.

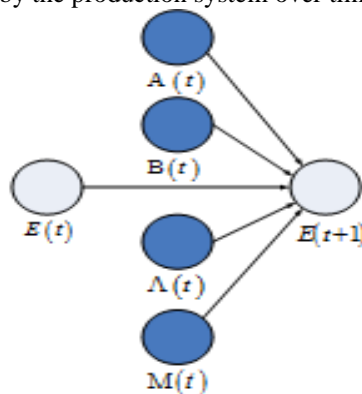


Fig. 7. Transition to MSS model of DBN type

$E(t)$, $E(t+1)$ respectively denote the state of system at time t , the state of system at time $t+1$.

$A(t)$ and $B(t)$ respectively denote vectors consisted of α_i and β_j with $j = 1, 2, 3, \dots, n$.

$\Lambda(t)$ and $M(t)$ respectively denote the vectors formed of λ_i and μ_i with $j = 1, 2, 3, \dots, n$

$$E(t) = \begin{pmatrix} e_1(t) \\ e_2(t) \\ \dots \\ e_n(t) \end{pmatrix}; E(t+1) = \begin{pmatrix} e_1(t+1) \\ e_2(t+1) \\ \dots \\ e_n(t+1) \end{pmatrix}$$

$$A(t) = \begin{pmatrix} \alpha_1(t) \\ \alpha_2(t) \\ \dots \\ \alpha_n(t) \end{pmatrix}; B(t) = \begin{pmatrix} \beta_1(t) \\ \beta_2(t) \\ \dots \\ \beta_n(t) \end{pmatrix}$$

$$\Lambda(t) = \begin{pmatrix} \lambda_1(t) \\ \lambda_2(t) \\ \dots \\ \lambda_n(t) \end{pmatrix}; M(t) = \begin{pmatrix} \mu_1(t) \\ \mu_2(t) \\ \dots \\ \mu_n(t) \end{pmatrix}$$

Structure of multi-state Bayesian model

We want a dynamic model more compact and simple which in addition has the advantages of Markov model. We integrate a decision node D for our study.

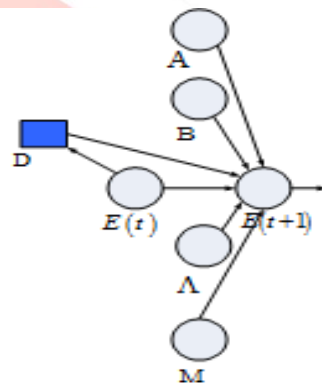


Fig. 8. Basic multi-state dynamic bayesian model

To fill in the conditional probability tables of our DBN structure, we use the Chapman-Kolmogorov equations to determine the transitions probability of system states.

$$P_1^{(k+1)} = (1 - \alpha_1^{(k)} - \lambda_1^{(k)})P_1^{(k)} + \mu_1^{(k)}P_2^{(k)} + x_1^{(k)}\beta_1^{(k)}P_{2d-1}^{(k)}$$

$$\forall j \in \{2, \dots, d-1\}: P_{2j-1}^{(k+1)} = (1 - \alpha_j^{(k)} - \lambda_j^{(k)})P_{2j-1}^{(k)} + \mu_j^{(k)}P_{2j}^{(k)} + \alpha_{j-1}^{(k)}P_{2j-3}^{(k)} + x_j^{(k)}\beta_j^{(k)}P_{2d-1}^{(k)}$$

$$P_{2d-1}^{(k+1)} = \left(1 - \alpha_d^{(k)} - \lambda_d^{(k)} - \sum_{j=1}^{d-1} x_j^{(k)}\beta_j^{(k)}\right)P_{2d-1}^{(k)} + \mu_d^{(k)}P_{2d}^{(k)} + \alpha_{d-1}^{(k)}P_{2d-3}^{(k)} \tag{3}$$

$$P_{2d+1}^{(k+1)} = (1 - \alpha_{d+1}^{(k)})P_{2d+1}^{(k)} + \alpha_d^{(k)}P_{2d-1}^{(k)}$$

$$\forall j \in \{2, \dots, d-1\}: P_{2d+j}^{(k+1)} = (1 - \alpha_{d+j}^{(k)})P_{2d+j}^{(k)} + \alpha_{d+j-1}^{(k)}P_{2d+j-1}^{(k)}$$

$$P_{2d+m}^{(k+1)} = P_{2d+m}^{(k)} + \alpha_{d+m-1}^{(k)}P_{2d+m-1}^{(k)}$$

$$P_1^{(0)} = 1, P_j^{(0)} = 0; \forall j \in \{2, \dots, n\}$$

$$\sum_{i=1}^n P_i^{(k)} = 1; 0 \leq t \leq T$$

In the next subsection, we include in our basic model the performance indicators.

Bayesian model with performance indicators

We consider in our approach the following performance indicators:

The availability of Multi-State System is the probability of being in an acceptable state of operation at time t :

$$A_v(t) = \sum_{j=1}^d P_{2j-1} \tag{4}$$

The states (2j-1) correspond to degraded states of the system.
 The maintenance cost model is:

$$C_{\text{sys}} = C_{\text{unav}} + C_{\text{degraded states}} + C_{\text{failure states}} \tag{5}$$

$$C_{\text{unav}} = \sum_{i=1}^{2d+m} \text{production penalty} \times P(E(t) = i) \tag{6}$$

$$C_{\text{degraded states}} = \sum_{j=2}^d x_D \cdot P_{2j-1} \tag{7}$$

$$C_{\text{failure states}} = \sum_{j=1}^d c_{\text{act}}(2j) \cdot p_{2j} \tag{8}$$

Where:

C_{sys} : Maintenance cost of system

C_{unav} : cost for unavailability of system

$C_{\text{degraded states}}$: Cost associated with degraded states

$C_{\text{failure states}}$: Cost associated with failure states

$C_{\text{act}}(2j)$: Cost associated with maintenance actions at the state 2j

Production penalty: penalty associated with the unavailability of the state i.

To follow the evolution of a given system, we consider a decision variable of preventive maintenance policy called

$$x_D : \begin{cases} 1 & \text{preventive maintenance action} \\ 0 & \text{no preventive maintenance action} \end{cases}$$

We integrate, with our structure some performance indicators of DBN such as the maintenance cost and availability (Fig.9).

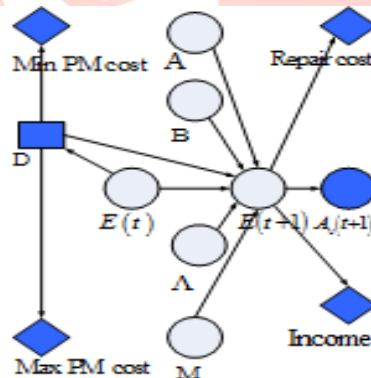


Fig. 9. Multi-state Bayesian model with indicators

VI. SIMULATION

It is considered that the transition parameters are constant and their values are taken, in Table2. In our study for the overall case of maintenance actions learning, we used the following learning algorithm parameters by reinforcement:

- a discount factor : 0.99
- a learning rate : 0.25
- an initial exploration rate : 0.50

To better analyze and get the optimal preventive maintenance level, we set here three ways:

- no preventive maintenance
- minimal preventive maintenance
- maximal preventive maintenance

The D decision node imposes the one of maintenance levels cited above and a learning algorithm to make a good decision among the terms at each iteration and the occurrence of the state 5.

Table 2. Transition parameters

α_1	α_2	α_3	λ_1	λ_2	λ_3	μ_1	μ_2	μ_3
0.03	0.05	0.07	0.005	0.008	0.01	0.01	0.02	0.04

Then, given the Table.2 and Chapman-Kolmogorov equations, The probability distributions of the various nodes of our DBN are calculated and put in their probability tables (CPT).

The simulation is made over a period of five years, with two teams working 17600 hours i.e. eight-hour working day during 20 working days in the month. We consider a system with six states (Fig.10)

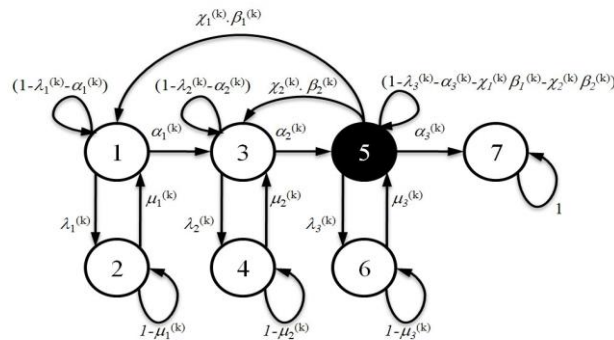


Fig. 10. Markov graph with constant parameters

Case 1: Simulation without learning according to income

The simulation studies, gave us about 35.086 % of availability with an average income of 2457 euro (Fig. 11).

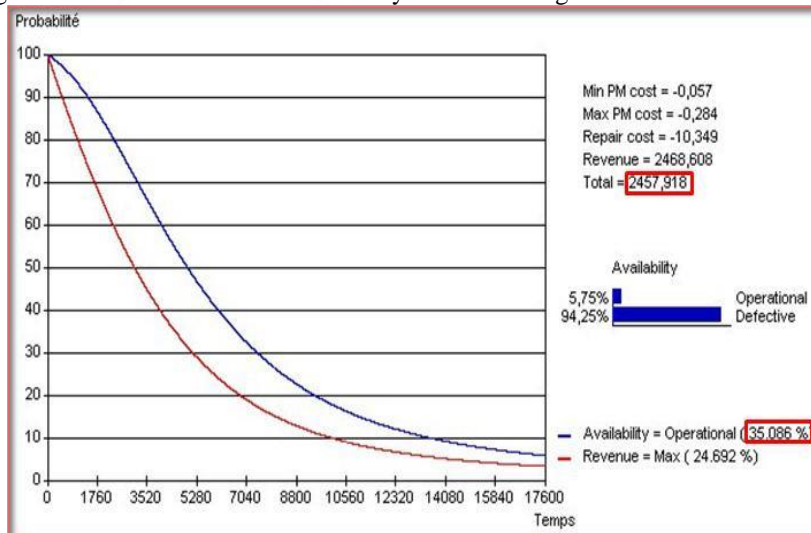


Fig. 11. Simulation curve

Case 2: Simulation with learning according to income

We note by learning on preventive maintenance (Fig.12), the system studied has about an availability of 37.692% against 35.086 % in the previous without learning (case1) and an average income of 2685.637 euro against 2468.608 euro in the same case 1 in the simulation (Fig. 11). So an increase of 2.61% in availability compared to simulation without learning.

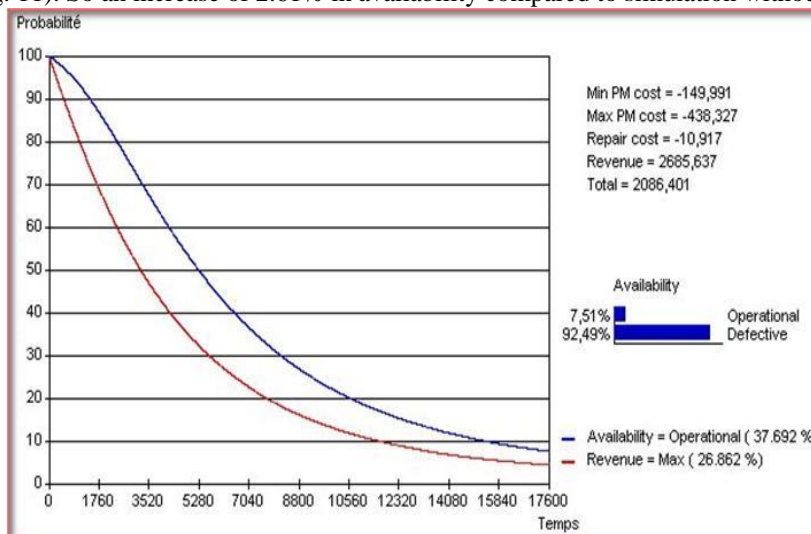


Fig. 12. Simulation curve: learning on preventive maintenance.

We give here, from the node D, an example about the best choice of maintenance according to time (Fig.13).

Time	Etat (t)	D1_pas_de...	D2_PM_Max	D3_PM_Min
1	E1	-839073,270	-875008,688	-875684,265
	E2	-572018,963	-566141,272	-566840,568
	E3	-790927,660	-768124,221	-768366,901
	E4	-421731,619	-419381,392	-419869,135
	E5	-430336,250	-419555,341	-419358,694
	E6	-92053,438	-91697,305	-91781,055
	E7	-3954282,129	-3950277,149	-3960627,200
2	E1	-837357,757	-875395,398	-874721,523
	E2	-571961,346	-566265,078	-566085,330
	E3	-791087,395	-767989,977	-767851,723
	E4	-421675,288	-419478,876	-419356,135
	E5	-430584,560	-419073,094	-419255,299
	E6	-92043,638	-91715,077	-91690,901
	E7	-3952743,847	-3952361,419	-3950540,228
3	E1	-837601,559	-875472,852	-874660,360
	E2	-572041,396	-566252,588	-566092,659
	E3	-791122,349	-767990,017	-767867,295
	E4	-421729,187	-419469,333	-419363,278
	E5	-430688,802	-419013,259	-419236,894
	E6	-92053,117	-91713,415	-91692,048
	E7	-3953788,419	-3952154,979	-3950748,502
	E1	-837907,295	-875455,179	-874628,250
	E2	-572127,134	-566233,462	-566101,083
	E3	-791147,505	-768016,363	-767871,023

Fig. 13. Node D and choices of maintenance policy

Case 3: Simulation without learning according to availability

Based on inputs (transition parameters, maintenance costs), the aim is to know about the optimal preventive maintenance level without learning and by optimizing the availability system.

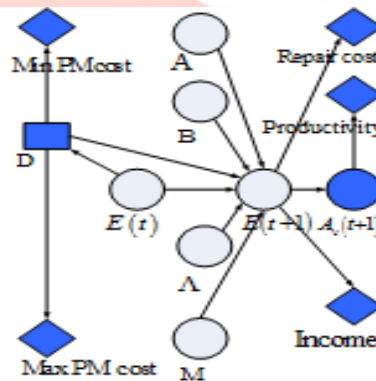


Fig. 14. Multi-state Bayesian model with more indicators.

We obtained without learning (fig.15), the system studied has about an average income of 999.981 euro and with an availability of 35,085% .

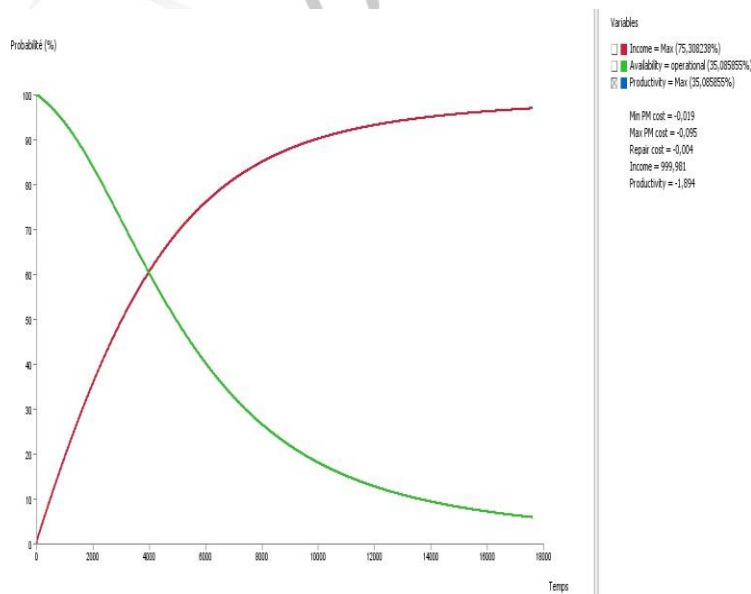


Fig. 15. Simulation curve without learning according to availability

Case 4: Simulation with learning according to availability

From the given inputs (transition parameters, maintenance costs), emerge the strong desire to know about the optimal preventive maintenance level by optimizing the availability of the system. Our system study offer us by learning (fig.16) about an availability of 41,32% and with average income of 666.705 euro.

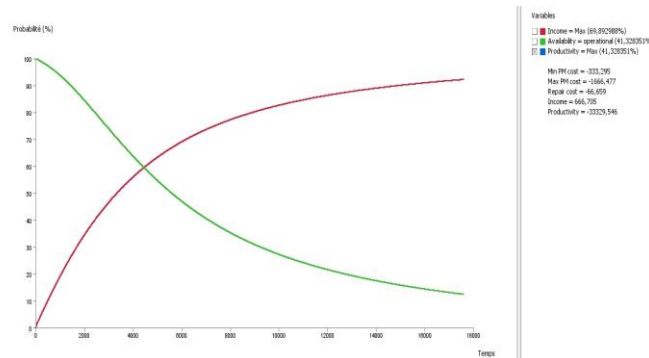


Fig. 16. Simulation curve with learning on preventive maintenance according to availability.

We note by learning on preventive maintenance (Fig.16), the system studied has about an availability of 41.328 % against 35.085% in the previous case without learning (case 3) and an average income of 666.705 euro against 999.981 euro in the same case 3 in the simulation (Fig. 15). So an increase of 6.243 % in availability compared to simulation without learning of case 3. We give a summary of test case in the table 3.

Table 3. Summary of test case for simulation

Test cases	Case1	Case2	Case3	Case4
income	2468.608	2685.637	999.981	666.705
availability	35.086	37.692	35.085	41.328

In our simulation with the learning on preventive maintenance, we noticed that the availability increases compared to the simulation without learning. Also, we saw that a decrease in the income which is presumably due the increase of the availability of system.

VII. CONCLUSIONS

This paper suggests an approach for the steering of multi-state systems through a real dynamic bayesian model. Our proposed approach allows to assess the performance and to follow the evolution of the behavior of multi-state systems subject to a variable maintenance policy. For, we first present an updated review of multi-state systems. Second we propose a way to categorize it. Third, we show how to conduct an assessment of the availability and the maintenance cost model.

Our formalism is based on stochastic processes such as Markov chain to model the dysfunctional behavior of the production systems in time by the DBN.

A simulation study made up of four test cases is carried out for the duration of five years to know the optimal maintenance policy according to time and we compared the obtained results in order to deduce the compromise between the different indicators of system performance.

The approach can be applied by the manufacturers subject to variability of the maintenance policy and/or de production.

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