

# A Review on Stresses Induced During Welding Process

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**Abstract** - Welding is one of the most used methods for joining metals. This method has largely developed by experiments, i.e. trial and error. The problems of distortion and residual stresses of a structure is due to welding is important to control. Thermal energy applied results in irreversible elastic-plastic deformation and consequently gives rise to the residual stresses in and around fusion zone and heat affected zone. It is well established fact that structural integrity of components is substantially affected by the residual stresses when subjected to thermal and structural loads. The aim of this paper is to study the transient stress field and residual stress field which are generated during welding process, study behaviour and factors affecting these stress fields.

**Index Terms** - Thermal strain, Welding Stresses

## I. INTRODUCTION

This Welding is widely used in automotive industries to assemble various products, also used in construction, ship building, steel bridges and pressure vessels. The advantage of welding is as a joining process, include high joint efficiency, simple set up and low fabrication cost.

The industry is actively considering a number of alternate welding technologies that would enable the increased use of lightweight and high performance materials. In the process of welding, the high temperature differences result in large temperature strains, which affect the distribution of the contact pressure between structural components.

The welding process causes a highly non-uniform heating of the parts being joined. Areas of the work piece close to the welding arc are heated up to several thousand degree Celsius (depending on the welding process), and then subsequently cooled down, conducting the heat further to the bulk of the body. The local heating and subsequent cooling induce volumetric changes producing transient and residual stresses and deformation.

Welding stresses and deformation are closely related phenomena. During heating and cooling thermal strain occur in the weld and adjacent areas. The strains produced during the heating stage of welding are always accompanied by plastic deformation of the metal. The stresses resulting from these strains combine and react to produce internal forces that cause a variety of welding distortions.

Residual deformations introduce severe problems in assembling of the welded structure and reduce its quality. Distorted shape and dimensions reduce the usefulness of the structure. The residual stresses have a strong influence on weld deformation, fatigue strength, fracture toughness and buckling strength. Thus, it is important to evaluate the residual stresses due to welding. However evaluating residual stresses associated to a welded joint is extremely complicated. Difficulty in determining these stresses is emphasized by the thermal transient, by the variation of the thermal and mechanical properties of the material with the temperature and by the non linear heat losses.

## II. BASIC EQUATIONS

One of the basic equations of the heat transfer analysis is Fourier's law of heat conduction. It represents the heat flow density  $q$  [ $J.m^{-2}.s^{-1}$ ] is depend on the temperature gradient  $\partial t / \partial n$  [ $K.m^{-1}$ ].

$$q = -\lambda \frac{\partial t}{\partial n} \quad 1$$

Where  $\lambda$  is the coefficient of thermal conductivity [ $J.m^{-1}.s^{-1}.K^{-1}$ ]. It maps the fact that the heat propagates in a solid from the hot to the colder areas.

The next equation should be mentioned as a main principle of heat transfer analysis is the field equation of heat conduction. Physically it associates the spatial and time dependent temperature distribution.

$$c\rho \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \lambda x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( \lambda z \frac{\partial T}{\partial z} \right) + q_2 \quad 2$$

Where,  $c$  is the mass-specific heat capacity, [ $J.kg^{-1}.K^{-1}$ ];  $\rho$  is the material density, [ $kg.m^{-3}$ ] and  $q_2$  is the volumetric density of the heat source, [ $W.m^{-3}$ ].

This equation can be easily derived based on Fourier's law of heat conduction and the law of energy conservation. Because in general case the  $c_p$ ,  $\lambda_x$ ,  $\lambda_y$ ,  $\lambda_z$  characteristics depend on temperature and is non-linear. The method of analytical solution forces us to ignore then temperature dependence of the above listed factors (the heat flow problems for welding applications are accessible to linearization and simplifications without significant loss of accuracy). This assumption leads to a simpler linearized form of the field equation of heat conduction.

$$\frac{\partial T}{\partial t} = \frac{\lambda}{c_p} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{q^2}{c_p} = \alpha \nabla^2 T + \frac{q^2}{c_p}$$

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Where  $\alpha = \frac{\lambda}{c_p}$  is the thermal diffusivity, [ $m^2.s$ ].

### III. WELDING STRESSES

Stresses arising during the welding process are referred to as internal or locked-in Stresses. Internal stresses are those which exist in a body without external forces applied. Internal stresses also subdivide into macro and micro-stresses (first, second, and third order). First order residual stress, extend over macroscopic areas and is the averaged stress over a volume with several material grains. Second order residual stress, acts between adjacent grains. It is averaged within each grain. Third order residual stress, acts on the inter-atomic level.

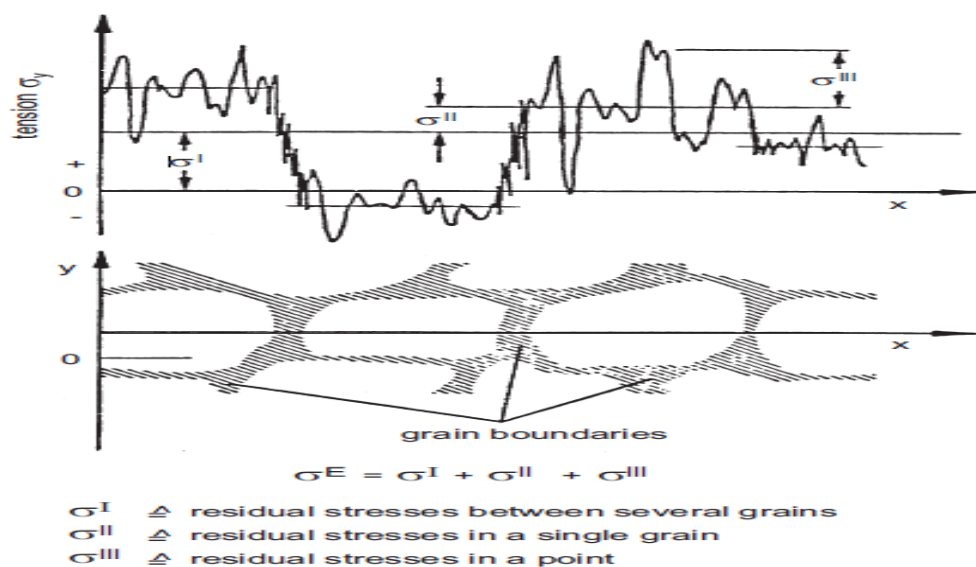


Fig. 1 Stresses of first, second and third order.

Welding stresses can be classified by their characteristics

1. Lifetime
2. Direction
3. Origin of

According to the first characteristic, welding stresses can be temporary or residual. The temporary stresses do exist only in a specific moment of the non-stationary process of heating and cooling. The residual stresses can be found after the whole process of welding is completed and structure is cooled down to the room temperature. Directionally the welding stresses subdivide into longitudinal (parallel to the welding direction) and transversal (perpendicular to the weld seam).

By origins the welding stresses are subdivided into

1. Thermal stress (caused by non-uniform temperature distribution)
2. Stresses caused by the plastic deformation of the metal
3. Stresses caused by phase transformations

Thermal stresses vanish after temperature equalization. Phase-transformation stresses may appear during welding of some alloyed steels. In processing low-alloyed structural steels, phase transformation occurs at elevated temperatures. The material, being soft, accommodates volume change caused by phase transformation without significant change in the stress development process. Stresses caused by plastic deformation almost always exist in the areas close to the weld and weld seam itself.

### IV. CAUSES OF WELDING STRESSES

The welding process causes a highly non-uniform heating of the parts being joined. Areas close to the weld arc are heated up to several thousand degrees Celsius, and then cooled down, the heat being conducted to the bulk of the body. The local heating and subsequent cooling induces volumetric changes producing temporary and residual stresses and deformation. Let us consider the

body to be composed of many equal small cubic elements. The process of heating these elements evenly will lead all the elements to uniform expansion in all spatial directions. Hence, all the elements will have the same size. It is possible to join these cubes and get a solid body, and no stresses will be induced in the body. But, if the heating is non-uniform, then each element tries to expand proportionally to the temperature rise  $\Delta T$  in element. In this situation the elements have different size, and it is not possible to join such cubes into one solid body. At the same time, the body is continuous and each element restrains the free expansion of the neighbouring elements. As a result stress is build up. The neighbours expanding differently act on the element in a different way. As a result of this, the lengths of the element edges change to a different degree, and angles at the vertices vary as well.

In other words, we get complex stresses both in the element and in the body. If, during heating, all the elements were stressed elastically, then, after cooling, the body will return to its initial stress-free condition. If, during heating, the element was deformed plastically, then, after cooling, it tends to change dimensions proportionally to the amount of the plastic deformation. All the elements now have different size and cannot be reassembled into a solid body without some changes in their stress- and deformation state. As a result, residual stresses and deformation form in the body.

## V. TRANSIENT STRESS FIELD

### 1. Longitudinal stress development during the welding process

As a starting point a typical thermal cycle can be taken (Fig. 2 low right corner). To monitor the stress development at a point, an elementary volume element should be chosen. Let us consider the case of thin plate welding (the through-thickness temperature gradient is negligible). The elementary element can be chosen as shown in Fig. 2 The temperature inside can be considered to be constant because the volume of the element is small.

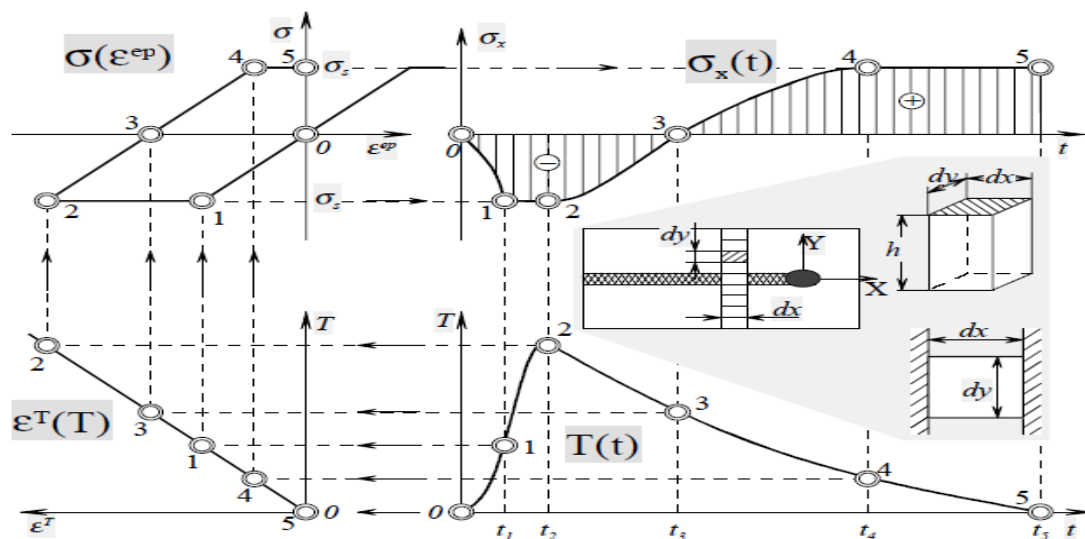


Fig. 2 Stresses in prismatic solid

Usually, a welded structure is stiff enough to keep the total deformation along the welding direction (in our case  $\epsilon_x$ ) significantly smaller than the unrestrained thermal deformation  $\epsilon^T$ . This state is true for the elements situated close to the weld seam. That is why it is normal to assume that the prism does not change its dimensions in the x direction (as shown in Fig. 2). At the same time, during the heating and cooling stages, longitudinal stresses  $\sigma_x$  develop in the elements.

For simplification let us consider the 1D-stressed case, and assume  $\sigma_y$  to be equal to zero (in the y-direction the element can be deformed stress-free). Summarizing, in the longitudinal direction,  $\sigma_x \neq 0$ ,  $\epsilon_x = 0$ , but in the transverse direction  $\sigma_y = 0$ ,  $\epsilon_y \neq 0$ .

To analyse the stress-cycles in the elements, data about the metal volume expansion due to the elevated temperatures (the dilatometric curve) and curves of material deformation (the stress-strain curves) are needed. Neglecting the structural changes in the material, the dilatometric curve can be approximated as a straight line (the thermal expansion coefficient  $\alpha$  is constant).

The set of stress-strain curves can, as a first approximation, be substituted by an idealized stress-strain curve (as shown in the upper left corner in Fig. 2). To systematize the analysis, the curve is scaled so that the elastic part of the curve would form  $45^\circ$  to the  $\epsilon^{ep}$ -axis (the elastic deformation and the corresponding stress are presented by the same length on the diagram).

The dilatometric curve  $\epsilon^T(T)$  is placed under the stress-strain diagram  $\sigma(\epsilon^{ep})$ . The thermal cycle  $T(t)$  is placed to the right of the  $\epsilon^T(T)$ . All the scales are kept co-ordinated. Now, the stress-cycle  $\sigma_x(t)$  can be plotted following the tracings in Fig.2 shown by the arrows.

For example, at the time  $t_1$  the temperature  $T_1$  is characterised by the point 1. From this point a horizontal line, characterising the thermal expansion corresponding to  $T_1$ , will determine the point 1 on the dilatometric curve. For the case of a rigid prism in the x direction, the  $\epsilon^T_1$  will determine the free (confined) thermal expansion. Extending vertically up onto the  $\sigma(\epsilon^{ep})$  diagram, another point 1 is obtained. This point is characterised by  $\sigma_x$  and  $\epsilon^e_x$  at  $t_1$  and coincident with the moment in time when the  $\sigma_x$  reaches the yielding limit  $\sigma_y$ .

Now the required point 1 on the stress-curve can be found by the intersection of the perpendiculars from the thermal cycle and the stress-strain curve. In the same manner the rest of the points characterising the stress-cycle can be found. The point 2 corresponds to the maximum temperature on the thermal cycle and the maximum of the plastic compressive strain.

After  $t_2$  the cooling process and, hence, the unloading start and last until  $t_3$ . At  $t_3$  the elastic stress and strain are both equal to zero. From  $t_3$  until  $t_4$  the elastic tensile strain is growing. At  $t_4$  the second plastic (but of the opposite sign) deformation begins. The time  $t_5$  corresponds to the completely cooled down state.

Fig. 3 gives a schematic representation of the temperature and the resulting longitudinal stress distributions that occur during welding. In this example a simple bead-on-plate case is analysed (see fig. 3a). The welding arc, which is moving along the x-axis with a speed  $v$ , is indicated by the arrow.

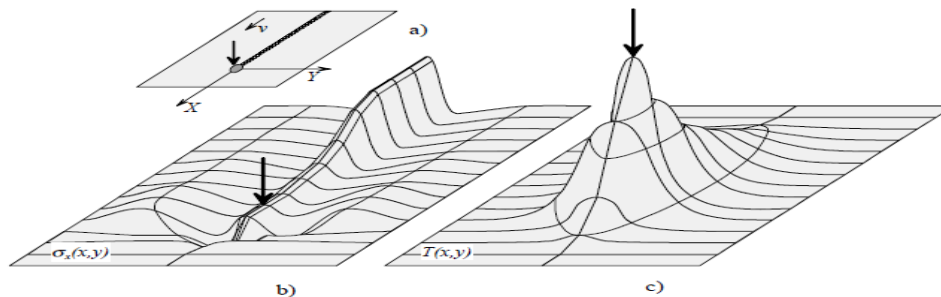


Fig. 3 Schematic representation of changes of temperature and stress during welding; (a) scheme of the welding process; (b) longitudinal stress distribution over the plate; (c) temperature distribution over the plate.

Far ahead from the heat source the temperature is constant and the stress is equal to zero in all the points. Moving in the negative direction of the x-axis, we reach the point where the temperature starts to rise. The points close to the weld line start to experience compression in the longitudinal direction. This deep fall changes to a fast rise of the longitudinal stress. The rate of stress change is proportional to the temperature gradient ahead of the source. It caused by the yielding point  $\sigma_y$  changing with temperature. As known, at elevated temperatures the material begins to soften. After some temperature the material reaches the stage when  $\sigma_y$  is almost zero. And so, the points situated close to the centreline reach the softening temperature, and climb up to a zero value of the longitudinal stress. Stresses in the regions a short distance from the arc are compressive, because the expansion of these areas is restrained by the surrounding metal where the temperature is lower. However, stresses in the areas further away from the weld arc are tensile and balanced by compressive stresses in the areas near the weld.

Going further, at some distance behind the welding arc, the temperature drops sufficiently for the material to be stiff enough to resist the deformation caused by the temperature change. Due to cooling the areas close to the weld contract and cause tensile stresses. After a certain time, the temperature change due to welding diminishes. High tensile longitudinal stresses (usually up to the yielding stress) are produced near the weld. In the regions further away from the weld, compressive stresses do exist.

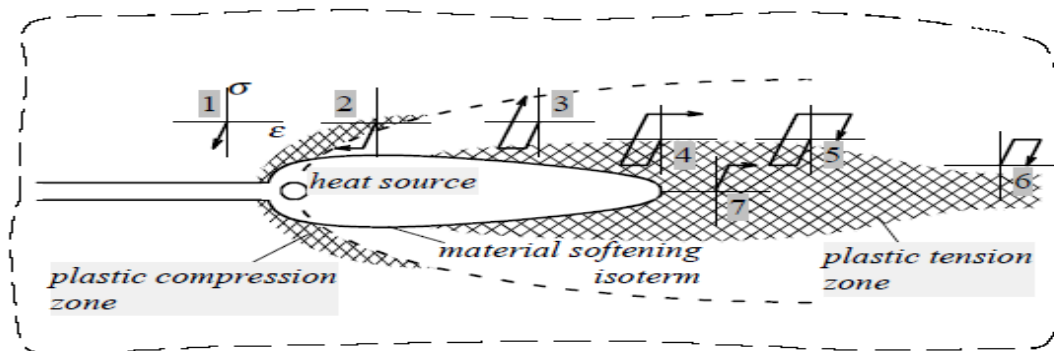


Fig. 4 Plastic compression and tension zones; local stress-strain cycles in quasi-stationary temperature field of the moving heat source

In Fig. 4 the scheme of plastic zone distribution for a case of quasi-stationary temperature field caused by a moving line heat source is presented. As mentioned above, the material reaching some temperature loses its strength. Beyond this temperature limit the material is almost free of stress because of the reduced yield limit at elevated temperatures. Furthermore, if the material has been subjected to some deformation or stress, after passing through this region the material becomes stress-free.

As an example the point 6 in Fig. 4 may be suggested. At first sight the schematic stress-strain cycle in point 6 should have looked like in point 5. But the difference is that material in point 6, after reaching some elastic and plastic compression, was "annealed" inside of the material softening isotherm. The parabola-like curve drawn as a broken line indicates the local temperature maximum. This line serves as a boundary between the area ahead of it, subjected to elastic or elasto-plastic compression, and area behind it, exposed to tensioning. The elastic unloading zone, corresponds to the segment 2-3 in Fig. 2, separates these areas.

Points 1, 2 and 3 are situated along one horizontal line. Therefore, they represent the consecutive stress development at a point lying at some distance from the weld centreline. First, the material is being exposed to elastic compression (point 1), and then, reaching the yield limit, the material undergoes plastic deformation (point 2), followed by elastic unloading (point 3). Point 7 has a



peculiar position. It lies on the weld centreline and the material in this point has being subjected only to elastic and then plastic tensioning. It is true because before the welding arc has passed over this point, the material in it did not exist, there was a groove.

## 2. Transverse stress development during the welding process

To demonstrate welding transverse stress development in a free plate, the results from a 3D FEM model are presented in Fig.5b. The evolution of stress was traced along the line transverse to the welding direction marked in Fig.5a. A first sight, the behaviour of the transverse stress is somewhat similar to the longitudinal stress behaviour, at least during the beginning of the cooling. Far ahead of the heat source, the stress is equal to zero because there is no disturbance factor, i.e. no temperature change. Approaching the heat source, we first meet a tensile stress hump, while the temperature is still equal to zero. Then follows an abrupt fall down to compression reaches the yield limit at slightly elevated temperatures. Then, with rising temperature and, hence, lower yield limit, the transverse stresses climb up to near zero values.

This highly compressed region right ahead of heat source explains the existence of the tensile hump. According to eq.

$$\int_A \sigma_x \cdot dA = 0, \quad \int_A \sigma_x \cdot y dA = 0, \quad \int_A \sigma_x \cdot z dA = 0$$

Where  $\sigma_x$  is the normal stress in the point with co-ordinates (y, z) in the cross-section area A.

The transverse stresses have to be in a self equilibrium state in the longitudinal section. So, the tension region has the function of sustaining the equilibrium against the unavoidable compression area around the hot elements of the structure trying to expand.

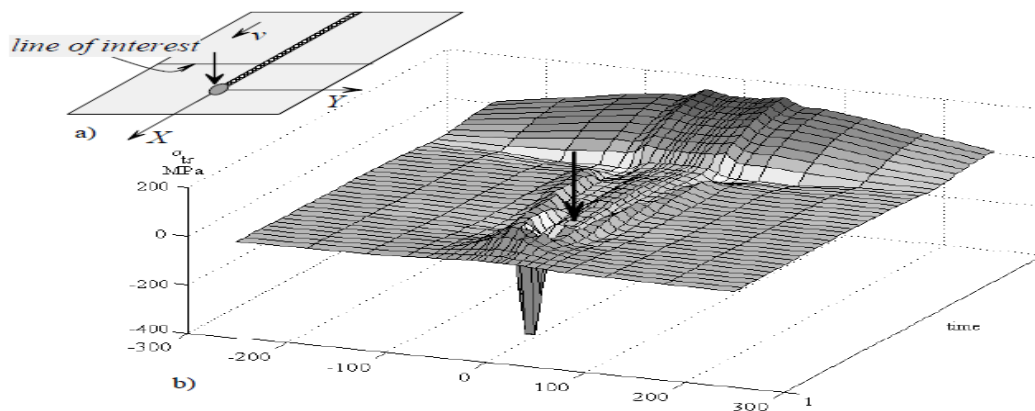


Fig. 5 Transverse stress evolution, (a) Scheme of the welding process; (b) Evolution of the transverse stress in a cross-section of the weld

Further along the time axis, at some distance behind the welding arc, the temperature drops sufficiently for the material to be stiff enough to resist the deformation caused by the temperature change. Due to cooling, the areas close to the weld contract and cause gradually growing tensile stresses.

## VI. RESIDUAL STRESS FIELD

### 1. Residual longitudinal stress formed due to the welding process

Maximum longitudinal residual stresses  $\sigma_x$  in welds are usually close to the yielding limit  $\sigma_y$ , gradually decreasing away from the weld axis in the plastic deformation zone, the longitudinal tensioning stresses then relax down to compression values in the adjacent areas. The residual stress distribution in a weld can differ from the one shown in Fig.6. Depending on alloying level of the weld- and base metal, the picture may change significantly. The cooling rate during welding process and the initial state of the steel can have a serious effect on the stress distribution.

Austenitic steels have thermal expansion coefficients  $\alpha$  greater than low-carbon steels. For these steels the softening happens at higher temperatures, compared to the low carbon steels. These facts evidences that austenitic steels undergo higher thermal stresses. This fact and the high level of plastic deformation cause metal strain hardening and, hence, generation of the longitudinal stresses higher than yielding limit for the non-deformed state of the material.

The behaviour of the longitudinal stresses in the cross-section of the weld is similar for austenitic and low-carbon steels.

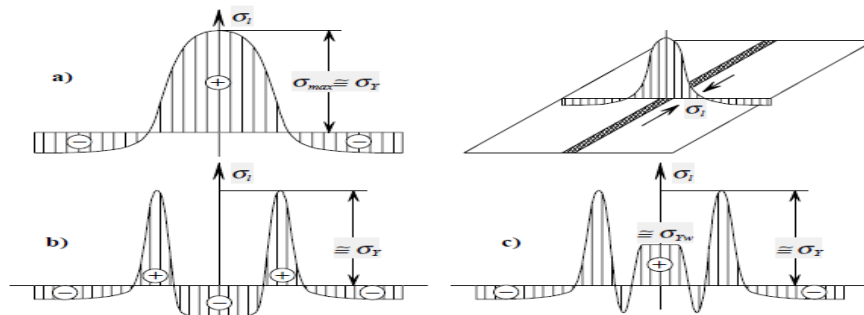


Fig. 6 Examples of weld-longitudinal stresses; (a) – for mild steel; (b) – for high alloy steel with martensitic filler metal; (c) – for high-alloy steel with austenitic filler metal.

The micro-structural changes in the materials during welding may modify the residual stress distribution radically. In Fig. 7b the dilatometric curve for a material with structural changes is presented.

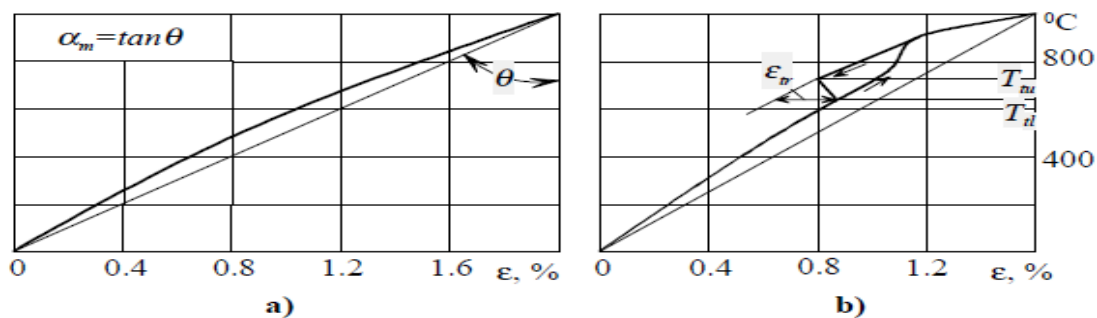


Fig. 7 Characteristic dilatometer curves; (a) – for austenitic steel; (b) – for steel with perlite microstructure

If structural changes during cooling happen at low temperatures, then the gradual contraction changes to a quick expansion. And the formed tensile stresses are reduced and turn into compression. Subsequent cooling produces further elongation of the material close to the weld seam and may give rise to tensile stresses. The residual stress distribution depends on the transformation strain  $\varepsilon_{tr}$  and the lower transformation temperature  $T_l$ . If the cooling rate is high enough to result in formation of martensite, then the areas close to the weld are in a compressed state (see Fig. 6b.).

The width of the plastic deformation zone depends on the welding parameters, the material properties and the stiffness of the structure. For this parameter the most important material properties are: the yield stress  $\sigma_y$ , the elastic modulus  $E$  and the thermal expansion coefficient  $\alpha$ . The greater the  $\sigma_y$  of the material, the narrower the plastic deformation zone, The greater  $E$  and  $\alpha$ , the wider the plastic deformation zone.

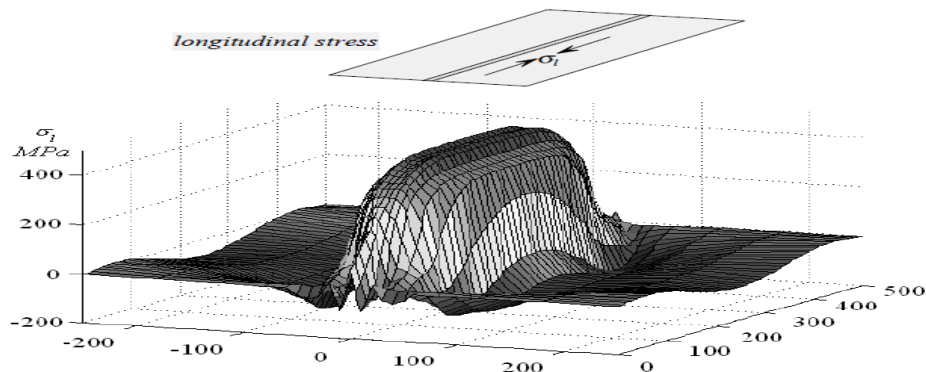


Fig. 8 3D representation of weld-longitudinal stress distribution.

The main welding parameters are the heat input per unit length  $q_w$  and the welding speed  $v$ . During welding of wide plates the higher value of  $q_w$ , the wider the plastic deformation zone. Increasing the welding speed with constant  $q_w$  and insignificant heat loss from the free surfaces leads to proportional growth of the isotherms. Increasing the welding speed with constant  $q_w$  results in reduction of the width of the plastic deformation zone. In Fig. 8 an example of 3D representation of longitudinal stress distribution is shown. It helps to create an overall notion about the complexity of the stress-state caused by the welding procedure. This distribution corresponds to the case when the material does not undergo a phase transformation at low temperature. In the middle of the plate it corresponds to Fig. 6a. The influence of the specimen length is depicted in Fig. 9. These data were acquired for welding of 25mm thick plates by two-pass SAW by DeGarmo et al.<sup>3</sup> These data cannot be taken as reference values for any welding parameters. It should be mentioned that the distance from the beginning or end of the weld, at which the longitudinal stress along the weld CL reaches its maximum value is close to the width of the plastic deformation zone.

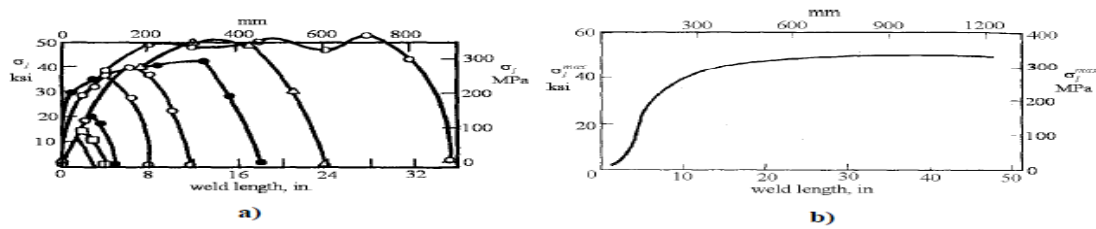


Fig. 9 Effect of length of weld on: (a) – longitudinal residual stress distribution; (b) – maximum longitudinal stress distribution

The reduction of end effects to acceptable levels relies on Saint-Venant's principle. For bodies extended in two or three dimensions the stress or strain due to loading on a small part of the body may be expected to diminish with distance on account of "geometrical divergence" whether or not the resultant is zero. This is also evident from Fig. 8. In summary it can be concluded that the shape of the graph in Fig. 9b depends on the welding parameters, the material properties and the stiffness of the structure. Longitudinal residual stresses start to grow at a distance from both ends of the weld. High tensile stresses exist in the central region of the welds. From Fig. 9a can be clearly seen that the peak stress in the central region increases with increasing weld length. It is evident that for the experiment under consideration, a weld longer than 450mm is needed to produce maximum residual tensile stresses in the longitudinal direction. Longitudinal residual stresses become uniform in the central region for welds longer than 450mm.

## 2. Residual transverse stress formed due to welding process

After the welded plates were cooled down, due to the transverse and longitudinal material shortening, residual transverse stresses arise in the structure. If the plates were welded in a free condition (without tack welds and additional clamps), then the transverse stresses are not too large. The greatest values are reached near the ends of the weld. Stresses there can be both compressive and tensile.

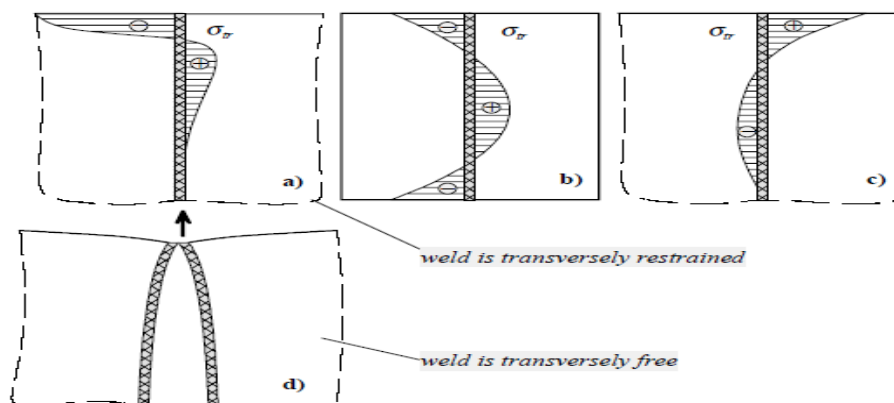


Fig. 10 Transverse residual stresses due to one-pass butt welding in: (a) – rapidly deposited weld in long plate; (b) – rapidly deposited weld in short plate; (c) – slowly deposited weld in long plate

A weld deposited instantaneously between two plates produces a gaping section in the middle of the weld length if the weld cools down without transverse restraint. The compressive stresses are initiated due to longitudinal shortening of the plastic deformation zone, and the plate edge tends to bend the way shown in Fig. 10d. Transverse compression arises at the weld ends with a change to transverse tension, when approaching the middle section of the weld length.

During welding of short and narrow plates with a high welding speed, the plate edges move towards each other during cooling. The longitudinal shortening tries to bend plates in the plane. As a result, the residual stresses illustrated in Fig. 10b are formed along the weld centerline.

If the welding speed is low enough for the weld metal to cool down to the temperature when the material is able to withstand loading (for mild steel the softening temperature  $T \approx 600^\circ\text{C}$ ), and it happens not so far behind the welding arc, then the end of the weld seam experiences tensile stresses as shown in Fig. 10c. The additional clamping has a serious effect on the residual transverse stress distribution in welds.

An example of 3D residual transverse stress distribution is given in Fig. 11. This distribution corresponds to the case of welding in small plates (see Fig. 10b). At the same time each plate is 250mm wide and 500mm long. So, the term "small" should be considered as complex term, combining not only geometrical dimensions but also welding parameters and even material properties.

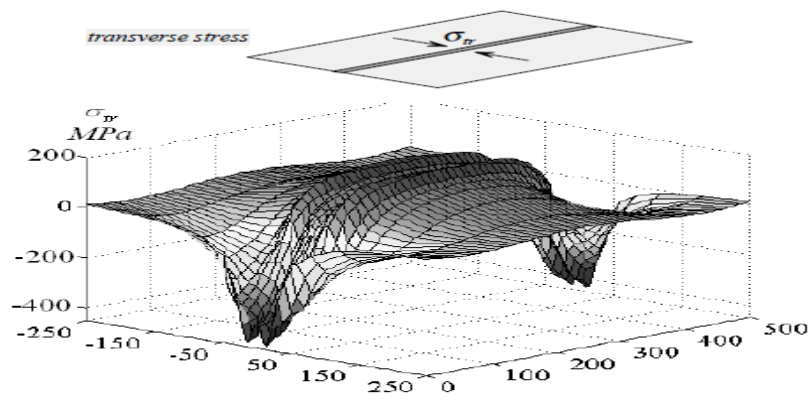


Fig.11 3D Representation of weld - Transverse stress distribution

From Fig. 11 can be seen that the residual transversal stresses are concentrated near the weld ends. It should be mentioned that transverse stresses in the through-thickness direction also exist in weldments. Their level is high enough in some cases of welding to pay serious attention to them.

An example for the case in Fig. 12.a can be a one-pass SAW used for joining two medium-thick plates or any kind of welding in thin plates. Such processes are usually characterized by a relatively wide plastically deformed zone. The material in the area close to the welding line is able to “breathe” in the through-thickness direction more or less freely. Hence, transverse stresses are low.

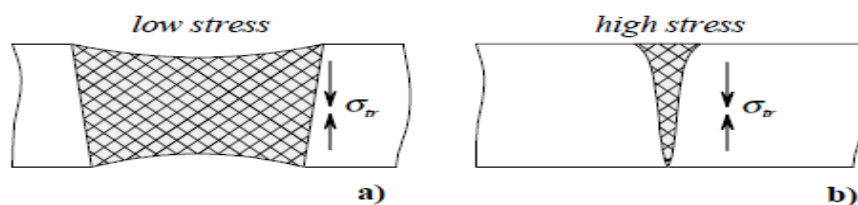


Fig.12 Transverse weld stress in through-thickness direction; (a) – with relatively wide plastically deformed zone; (b) – with narrow plastically deformed zone

To Fig. 12b correspond the case of highly concentrated electron-beam welding in thick and middle-thick plates. The metal of the narrow area close to the welding line is restricted in a through-thickness movement. And, in such a way, high residual stresses in the through-thickness direction are initiated.

## VII. CONCLUSION

Deformation and stresses generated during welding process are caused by non-uniform heating of parts being joined. The welding residual stresses and deformation are formed due to plastic deformation during the heating stage and subsequent cooling; longitudinal stress usually prevails over the transverse stress. Close to the end of the weld the residual transverse stress is substantial. Low temperature micro-structural transformation in the weld metal and heat affected zone can change the residual stress distribution significantly.

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