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The Importance of Gauss-Jordan Elimination Methods for Balancing Chemical Reaction Equation

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Abstract - This study shows Gauss-Jordan elimination method can help students to obtain the minimal positive integer that balance the number of atoms in reactant and products are conserved as many chemistry students cannot understand how to balance the equation of chemical reaction with maximum numbers of atoms or molecules appear as reactants and products. Hence, this problem can be treated with solving a system of linear equation by transforming its augmented matrix of the corresponding system to reduced row echelon form using successive process of elementary row operations that leads to find the solution for the minimal positive integer that balance the chemical reaction equation.

Key Word - Linear System, Gauss-Jordan Elimination, Row Operation, Chemical Reaction Equation

I. INTRODUCTION

Linear algebra is a cornerstone in undergraduate mathematical education. It develops a general language used by all scientists and is interdisciplinary in essence. One of the most frequently recurring practical problems in many fields of study such as mathematics, physics, biology, chemistry, economics, all phases of engineering , operations research, and the social sciences is that of solving a system of linear equations. Systems of linear equations play an important and motivating role in the subject of linear algebra. In fact, many problems in linear algebra reduce to finding the solution of a system of linear equations [1]. A study reveals that linear algebra at present is of growing importance in engineering research, science, frameworks, electrical networks, traffic flow, economics, statistics, technologies, and many others[2].

Particularly, in chemistry, one of the linear algebras techniques, namely, Gauss-Jordan elimination method can be used to solve one of chemistry's inevitable tasks of balancing chemical equations. A chemical equation is the symbolic representation of a chemical reaction in terms of chemical formulas. When the coefficients in a chemical equation are correctly given, the numbers of atoms of each element are equal on both sides of the arrow. The equation is then said to be balanced and it is preferable to write the coefficients so that they are the smallest whole numbers possible. That is, a chemical equation should be balanced follows from atomic theory (a chemical reaction involves simply a recombination of the atoms; none are destroyed and none are created) [3]. A study state that the substances taking part in a chemical reaction are represented by their molecular formulae, and their symbolic representation is termed as chemical equation [4]. Chemical equation therefore is an expression showing symbolic representation of the reactants and the products usually positioned on the left side and on right side in a particular chemical reactions [5].

Balancing of chemical reaction equations can be made much easier, especially for those who find it difficult, by moving the procedures toward the algorithmic and away from the heuristic. That is, a "step to step" procedure is simpler to master than is the haphazard hopping of inspection, even a highly refined inspection [6]. Moreover, the chemical equation for a reaction gives two important types of information: the nature of the reactants and products and the relative numbers of each. The relative numbers of reactants and products in a reaction are indicated by the coefficients in the balanced equation. The coefficients can be determined because we know that the same number of each type of atom must occur on both sides of the equation. Whenever you see an equation, you should ask yourself whether it is balanced. The principle that lies at the heart of the balancing process is that atoms are conserved in a chemical reaction. The same number of each type of atom must be found among the reactants and products. The formulas of the compounds must never be changed in balancing a chemical equation. That is, the subscripts in a formula cannot be changed, nor can atoms be added or subtracted from a formula. Most chemical reaction equations can be balanced by inspection, that is, by trial and error. It is always best to start with the most complicated molecules (those containing the greatest number of atoms) [7].

Many students typically become irritated with the inspection method, informally referred to as the trial-and-error method, when the student experiences a complication with applying it to more advanced problems that do not lend easily to an instantaneous or serendipitous solution. The second common teaching method involves applying linear algebra to solve for balanced chemical reactions with a system of linear equations [8]. However, there are many ways to find the solution of system of linear equation. Gauss-Jordan elimination is the common way used by students when the system of equations transformed into matrices. In a university, when solving systems of linear equations by using matrices, many teachers present a Gauss-Jordan elimination approach to row reducing matrices that can involve painfully tedious operations with fractions [9].

For chemists it is enough to find the minimal positive integer numbers of reactant and product must be equal during a chemical reaction. This study help chemistry students understand how to construct a homogenous system of linear equations whose solution provides suitable value to balance a chemical reaction equation using Gauss-Jordan elimination method.

II. BACKGROUND THEORETICAL CONCEPTS

System of Linear Equation

Systems of linear equations play an important and motivating role in the subject of linear algebra. In fact, many problems in linear algebra reduce to finding the solution of a system of linear equations.

Definition: A linear equation with n variables $x_1, x_2, x_3, \ldots, x_n$ has the form

$$a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_nx_n = b \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (1)$$

The coefficients a_1,a_2, a_3,…, a_n and the constant term b are real numbers. The number a_1 is the leading coefficient, and x_1 is the leading variable [10].

Note that linear equations have no products or roots of variables and no variables involved in trigonometric, exponential, or logarithmic functions.

Definition: A solution of a linear equation in n variables is a sequence of real numbers $s_1, s_2, \ldots, s_n$ arranged to satisfy the equation when you substitute the values $x_1 = s_1, x_2 = s_2, \ldots, x_n = s_n$ in the equation. The set of all solutions of a linear equation is called its solution set of the general solution of the system, and when you have found this set, you have solved the equation.

Definition: A system of m linear equations in n variables or unknowns is a set of m equations, each of which is linear in the same n variables. That is,

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3n}x_n &= b_3 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n &= b_m \end{aligned} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (2)$$

The system in equation (2) can be write in a matrix form $AX = b$ where A is an $m \times n$ matrix, called coefficient matrix, X is $n \times 1$ column matrix and b is $m \times 1$ column matrices represented by

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} \text{ and } b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_m \end{bmatrix}.$$

Definition: The matrix derived from the coefficients and constant terms of a system of linear equations is called the augmented matrix of the system.

Definition: A solution of a system of linear equations is a sequence of numbers s_1, s_2, s_3,…, s_n that is a solution of each of the linear equations in the system.

It is possible for a system of linear equations to have exactly one solution, infinitely many solutions, or no solution. In another case, we call a system of linear equations is consistent when it has at least one solution and inconsistent when it has no solution. Moreover, systems of linear equations also classified as homogeneous (when each of the constant terms is equal to zero) or non-homogeneous (when at least one of the constant terms is not equal to zero).

Remark: Homogeneous systems are always consistent, that is, it must have at least one solution. This is because all of the variables can be set equal to zero to satisfy all of the equations.

Gauss-Jordan Elimination Method

Definition:- An $m \times n$ matrix is said to be in reduced row-echelon form if it satisfies the following conditions:

(a) The rows (if any) consisting entirely of zeros are grouped together at the bottom of the matrix.
(b) The first nonzero entry from the left of a nonzero row is a leading 1.We will call this the pivotal entry for the row and the column where this appears the pivotal column.
(c) For each nonzero row, the leading one appears to the right and below any leading ones in preceding rows.
(d) If a column contains a leading one, then all other entries in that column are zero.

Definition: Two systems of linear equations are said to be equivalent if their solutions sets are identical, that is, if they have the same reduced row echelon form.

To convert a system of linear equation in to its row equivalent, we always must do the following elementary row operations with its notation.

a) Multiply a row by a nonzero number, denoted by cr_i.
b) Add a multiple of one row to another row, denoted by $r_i + cr_j$.
c) Swap any two rows, denoted by $r_i \leftrightarrow r_j$.

The method of solving a linear system by Gauss-Jordan Elimination is called an algorithm (a finite procedure, written in fixed symbolic vocabulary, governed by precise instructions). That is, the general procedure for Gauss-Jordan elimination method to solve a system of linear equations is described in the following steps.

Step 1: First, write the augmented matrix of the system.

Step 2: Next, use row operations to transform the augmented matrix in to the form called the reduced row echelon form (RREF).

Step 3: Finally, stop process in Step 2 if you obtain a row whose elements are all zeros except the last one on the right. In that case, the system is inconsistent and has no solutions. Otherwise, finish step 2 and read the solutions of the system from the final matrix. Note that try to choose row operations so that as few fractions as possible are carried through the computation. This makes calculation easier when working by hand.

III. RESEARCH METHOD

An empirical formula for a chemical reaction is an equation containing the minimal integer multiples of the reactants and products so that the number of atoms of each element agrees on both sides. Finding the empirical formula is called balancing the equation. Chemical reactions can be described by equations. The expressions on the left side are called the reactants, and those on the right side are the products, which are produced from the reaction of chemicals on the left. Unlike mathematical equations, the two sides are separated by an arrow, either ⟶, which indicates that the reactants form the products, or ⟷, which indicates a reversible equation: that is, once the products are formed, they begin to fom reactants.

A chemical equation is balanced, provided that the number of atoms of each type on the left is the same as the number of atom; of the corresponding type on the right. Here, we illustrate how to construct a homogeneous system of linear equations whose solution provides appropriate values to balance the atoms or ions in the reactants with those in the products. A solution of a chemical equation balance problem is a list of coefficients that appear on the various terms in the chemical equation. When a chemical equation is balanced, the number of atoms of each typee on the left side of the equation matches the number of corresponding atoms on the right side. The order of varius atoms is not important. Hence, we selecte by writing the elements in the order in which they first appear in the chemical equation, reading left to the right.

Balancing chemical reaction equation does not have a clear cut method of obtaining the coefficients of reactants and products in a chemical reaction equation. In chemical equation, sine neither an atom is created nor destroyed during the chemical reaction, the corresponding augmented matrix becomes a homogenous system as a result it has a unique solution by taking the smallest positive integers to be the coefficients of the atom or ions or molecules in chemical reaction equation.

This study describes how to balance the chemical reaction equation given in the following motivating two examples that are easily reached to undergraduate chemistry students by so called Gauss-Jordan method in a simple and concrete setting with an algorithm, or a systematic procedure side by side to give brief insight to the reader by replacing one system with an equivalent system (i.e., one with the same solution set) that is easier to solve. Since the existence of a solution is for the homogenous system is already clear, but the solution is not unique because there are free variables and thus each different choice of the free variables determines a different solution. If the coefficients in the equations are each multiplied by a fixed positve integer, the eqaution will remain balanced. So, there are many solutions to a chemical equation balanced problem. But, we need only the minimal integer multiples of the reactants and products so that the number of atoms of each element agrees on both sides.

Example 1: Use Gauss-Jordan elimination method to find the minimal positive integer values for the unknowns that will balance when sodium bicarbonate react with citric acid to give sodium citrate, water and carbon dioxide, that is, its chemical reaction equation can be written as $\mathrm{NaHCO_3 + C_6H_8O_7 \longrightarrow Na_3C_6H_5O_7 + H_2O + CO_2}$.

Solution: To balance this equation, we insert variables, multiplying the chemicals on the left and right to get an equation of the form

$$x_1\mathrm{NaHCO_3} + x_2\mathrm{C_6H_8O_7} \longrightarrow x_3\mathrm{Na_3C_6H_5O_7} + x_4\mathrm{H_2O} + x_5\mathrm{CO_2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (3)$$

Now we want to find the minimal positive integer values of x_1, x_2, x_3, x_4, and x_5 such that balances the number of Sodium(Na), Hydrogen(H), Carbon(C) and Oxygen(O) atoms on both sides of the linear equation below. Since this qualitative equation does not tell us how much of each reactant is needed and the relative amounts of the product are realized, the number of each atom in equation 3 must balance before and after the reaction which yields the following five linear equations corresponding to their atoms and we use Gauss-Jordan elimination method to solve the linear systems:

$$\begin{array}{ll}
x_1 = 3x_3 & \text{from balancing the number of Na atoms}\\
x_1 + 8x_2 = 5x_3 + 2x_4 & \text{from balancing the number of H atoms}\\
x_1 + 6x_2 = 6x_3 + x_5 & \text{from balancing the number of C atoms}\\
3x_1 + 7x_2 = 7x_3 + x_4 + 2x_5 & \text{from balancing the number of O atoms}
\end{array}$$

Rewriting the above systems of linear equation in standard form by bringing the x_3, x_4 and x_5 terms to the left side of each equation yields the following system of linear equation in five variables. Here, we describe the augmented matrix with its corresponding equivalent linear system side by side but elementary row operations are written underneath as follows.

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0\\ 1 & 8 & -5 & -2 & 0 & 0\\ 1 & 6 & -6 & 0 & -1 & 0\\ 3 & 7 & -7 & -1 & -2 & 0 \end{bmatrix} \quad \text{its corresponding linear system is} \quad \begin{array}{l} x_1 - 3x_3 = 0\\ x_1 + 8x_2 - 5x_3 - 2x_4 = 0\\ x_1 + 6x_2 - 6x_3 - x_5 = 0\\ 3x_1 + 7x_2 - 7x_3 - x_4 - 2x_5 = 0 \end{array}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0\\ 0 & 8 & -2 & -2 & 0 & 0\\ 0 & 6 & -3 & 0 & -1 & 0\\ 0 & 7 & 2 & -1 & -2 & 0 \end{bmatrix}}_{\substack{-r_1+r_2\to r_2\\ -r_1+r_3\to r_3\\ -3r_1+r_4\to r_4}} \quad \text{it is equivalent to} \quad \begin{array}{l} x_1 - 3x_3 = 0\\ 8x_2 - 2x_3 - 2x_4 = 0\\ 6x_2 - 3x_3 - x_5 = 0\\ 7x_2 + 2x_3 - x_4 - 2x_5 = 0 \end{array}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0\\ 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & 0 & 0\\ 0 & 6 & -3 & 0 & -1 & 0\\ 0 & 7 & 2 & -1 & -2 & 0 \end{bmatrix}}_{\frac{1}{8}r_2\to r_2} \quad \text{it is equivalent to} \quad \begin{array}{l} x_1 - 3x_3 = 0\\ x_2 - \frac{1}{4}x_3 - \frac{1}{4}x_4 = 0\\ 6x_2 - 3x_3 - x_5 = 0\\ 7x_2 + 2x_3 - x_4 - 2x_5 = 0 \end{array}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & 0 & 0 \\ 0 & 0 & -\frac{6}{4} & \frac{6}{4} & -1 & 0 \\ 0 & 0 & \frac{15}{4} & \frac{3}{4} & -2 & 0 \end{bmatrix}}_{\substack{-6r_2+r_3\to r_3 \\ -7r_2+r_4\to r_4}} \quad \text{it is equivalent to} \quad \begin{aligned} x_1 - 3x_3 &= 0 \\ x_2 - \tfrac{1}{4}x_3 - \tfrac{1}{4}x_4 &= 0 \\ -\tfrac{6}{4}x_3 + \tfrac{6}{4}x_4 - x_5 &= 0 \\ \tfrac{15}{4}x_3 + \tfrac{3}{4}x_4 - 2x_5 &= 0 \end{aligned}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & 0 & 0 \\ 0 & 0 & 1 & -1 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{15}{4} & \frac{3}{4} & -2 & 0 \end{bmatrix}}_{-\frac{4}{6}r_3\to r_3} \quad \text{it is equivalent to} \quad \begin{aligned} x_1 - 3x_3 &= 0 \\ x_2 - \tfrac{1}{4}x_3 - \tfrac{1}{4}x_4 &= 0 \\ x_3 - x_4 + \tfrac{2}{3}x_5 &= 0 \\ \tfrac{15}{4}x_3 + \tfrac{3}{4}x_4 - 2x_5 &= 0 \end{aligned}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & 0 & 0 \\ 0 & 0 & 1 & -1 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & \frac{9}{2} & -\frac{9}{2} & 0 \end{bmatrix}}_{-\frac{15}{4}r_3+r_4\to r_4} \quad \text{it is equivalent to} \quad \begin{aligned} x_1 - 3x_3 &= 0 \\ x_2 - \tfrac{1}{4}x_3 - \tfrac{1}{4}x_4 &= 0 \\ x_3 - x_4 + \tfrac{2}{3}x_5 &= 0 \\ \tfrac{9}{2}x_4 - \tfrac{9}{2}x_5 &= 0 \end{aligned}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & 0 & 0 \\ 0 & 0 & 1 & -1 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}}_{-\frac{2}{9}r_4\to r_4} \quad \text{it is equivalent to} \quad \begin{aligned} x_1 - 3x_3 &= 0 \\ x_2 - \tfrac{1}{4}x_3 - \tfrac{1}{4}x_4 &= 0 \\ x_3 - x_4 + \tfrac{2}{3}x_5 &= 0 \\ x_4 - x_5 &= 0 \end{aligned}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & \frac{-1}{4} & 0 & \frac{-1}{4} & 0 \\ 0 & 0 & 1 & 0 & \frac{-1}{3} & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}}_{\substack{\frac{1}{4}r_4+r_2\to r_2 \\ r_4+r_3\to r_3}} \quad \text{it is equivalent to} \quad \begin{aligned} x_1 - 3x_3 &= 0 \\ x_2 - \tfrac{1}{4}x_3 - \tfrac{1}{4}x_5 &= 0 \\ x_3 - \tfrac{1}{3}x_5 &= 0 \\ x_4 - x_5 &= 0 \end{aligned}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & \frac{-1}{3} & 0 \\ 0 & 0 & 1 & 0 & \frac{-1}{3} & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}}_{\substack{\frac{1}{4}r_3+r_2\to r_2 \\ 3r_3+r_1\to r_1}} \quad \text{it is equivalent to} \quad \begin{aligned} x_1 - x_5 &= 0 \\ x_2 - \tfrac{1}{3}x_5 &= 0 \\ x_3 - \tfrac{1}{3}x_5 &= 0 \\ x_4 - x_5 &= 0 \end{aligned}$$

Solving the system yields $x_1 = x_5$, $x_2 = \frac{1}{3}x_5$, $x_3 = \frac{1}{3}x_5$ and $x_4 = x_5$. Since x_5 a free variable we can be assigned any real number and we are dealing with elements or atoms or molecules, it is suitable to select values such that all the variables are become natural numbers. If we take $x_5 = 3$, we obtain $x_1 = x_4 = x_5 = 3$, and $x_2 = x_3 = 1$. Therefore, the empirical formula for the reaction is

$$3NaHCO_3 + C_6H_8O_7 \longrightarrow Na_3C_6H_5O_7 + 3H_2O + 3CO_2$$

Example2: Use the Gauss-Jordan elimination method to find the minimal positive integer values for the unknowns that will balance the given chemical equation:

$$MnSO_4 + NaBiO_3 + H_2SO_4 \longrightarrow Na_2MnO_4 + Bi_2(SO_4)_3 + H_2O + NaSO_4 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Solution: To balance this equation, we insert unknowns, multiplying the chemicals on the left and right to get an equation of the form

$$x_1MnSO_4 + x_2NaBiO_3 + x_3H_2SO_4 \longrightarrow x_4Na_2MnO_4 + x_5Bi_2(SO_4)_3 + x_6H_2O + x_7NaSO_4 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Now we want to find the minimal positive integer values of x_1, x_2, x_3, x_4, x_5, x_6 and x_7 such that balances the number of Manganese(Mn), Bismuth(Bi), Hydrogen(H), Sodium(Na), Sulfur(S) and Oxygen (O) atoms on both sides of the linear equation above. Since this qualitative equation does not tell us how much of each reactant is needed and the relative amounts of the product are realized, the number of each atom in equation (5) must balance before and after the reaction which yields the following six linear equations corresponding to their atoms with seven unknowns and we use Gauss-Jordan elimination method to solve this linear systems as follows:

$x_1 = x_4$	from balancing the number of Mn atoms
$x_2 = 2x_5$	from balancing the number of Bi atoms
$x_3 = 2x_6$	from balancing the number of H atoms
$x_2 = 2x_4 + 2x_7$	from balancing the number of Na atoms
$x_1 + x_3 = 3x_5 + x_7$	from balancing the number of S atoms

$$4x_1 + 3x_2 + 4x_3 = 4x_4 + 12x_5 + x_6 + 4x_7$$ from balancing the number of O atoms

Rewriting the above systems of linear equation in standard form by bringing the x_4, x_5, x_6 and x_7 terms to the left side of each equation yields the following system of linear equation in seven variables.

Here, we describe the augmented matrix with its corresponding equivalent linear system side by side but elementary row operations are written underneath as follows.

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & -2 & 0 \\ 1 & 0 & 1 & 0 & -3 & 0 & -1 & 0 \\ 4 & 3 & 4 & -4 & -12 & -1 & -4 & 0 \end{bmatrix}$$

its corresponding system is

$$\begin{aligned} x_1 - x_4 &= 0 \\ x_2 - 2x_5 &= 0 \\ 2x_3 - 2x_6 &= 0 \\ x_2 - x_4 - 2x_7 &= 0 \\ x_1 + x_3 - 3x_5 - x_7 &= 0 \\ 4x_1 + 3x_2 + 4x_3 - 4x_4 - 12x_5 - x_6 - 4x_7 &= 0 \end{aligned}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & -2 & 0 \\ 0 & 0 & 1 & 1 & -3 & 0 & -1 & 0 \\ 0 & 3 & 4 & 0 & -12 & -1 & -4 & 0 \end{bmatrix}}_{\substack{-r_1+r_5\to r_5 \\ -4r_1+r_6\to r_6}}$$

it is equivalent to

$$\begin{aligned} x_1 - x_4 &= 0 \\ x_2 - 2x_5 &= 0 \\ 2x_3 - 2x_6 &= 0 \\ x_2 - x_4 - 2x_7 &= 0 \\ +x_3 - 3x_5 - x_7 &= 0 \\ 3x_2 + 4x_3 - 4x_4 - 12x_5 - x_6 - 4x_7 &= 0 \end{aligned}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & 0 & -2 & 0 \\ 0 & 0 & 1 & 1 & -3 & 0 & -1 & 0 \\ 0 & 0 & 4 & 0 & -6 & -1 & -4 & 0 \end{bmatrix}}_{\substack{-r_2+r_4\to r_4 \\ -3r_2+r_6\to r_6}}$$

it is equivalent to

$$\begin{aligned} x_1 - x_4 &= 0 \\ x_2 - 2x_5 &= 0 \\ 2x_3 - 2x_6 &= 0 \\ -x_4 + 2x_5 - 2x_7 &= 0 \\ x_3 + x_4 - 3x_5 - x_7 &= 0 \\ 4x_3 - 6x_5 - x_6 - 4x_7 &= 0 \end{aligned}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & 0 & -2 & 0 \\ 0 & 0 & 1 & 1 & -3 & 0 & -1 & 0 \\ 0 & 0 & 4 & 0 & -6 & -1 & -4 & 0 \end{bmatrix}}_{\frac{1}{2}r_3\to r_3}$$

it is equivalent to

$$\begin{aligned} x_1 - x_4 &= 0 \\ x_2 - 2x_5 &= 0 \\ x_3 - x_6 &= 0 \\ -x_4 + 2x_5 - 2x_7 &= 0 \\ x_3 + x_4 - 3x_5 - x_7 &= 0 \\ 4x_3 - 6x_5 - x_6 - 4x_7 &= 0 \end{aligned}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 & -3 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & -6 & 3 & -4 & 0 \end{bmatrix}}_{\substack{-r_3+r_5\to r_5 \\ -4r_3+r_6\to r_6}}$$

it is equivalent to

$$\begin{aligned} x_1 - x_4 &= 0 \\ x_2 - 2x_5 &= 0 \\ x_3 - x_6 &= 0 \\ -x_4 + 2x_5 - 2x_7 &= 0 \\ x_4 - x_5 + x_6 - x_7 &= 0 \\ 6x_5 + 3x_6 - 4x_7 &= 0 \end{aligned}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -3 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & -6 & 3 & -4 & 0 \end{bmatrix}}_{r_4\leftrightarrow r_5}$$

it is equivalent to

$$\begin{aligned} x_1 - x_4 &= 0 \\ x_2 - 2x_5 &= 0 \\ x_3 - x_6 &= 0 \\ x_4 - 3x_5 + x_6 - x_7 &= 0 \\ -x_4 + 2x_5 - 2x_7 &= 0 \\ 6x_5 + 3x_6 - 4x_7 &= 0 \end{aligned}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -3 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 & -6 & 3 & -4 & 0 \end{bmatrix}}_{r_4+r_5\to r_5}$$

it is equivalent to

$$\begin{aligned} x_1 - x_4 &= 0 \\ x_2 - 2x_5 &= 0 \\ x_3 - x_6 &= 0 \\ x_4 - 3x_5 + x_6 - x_7 &= 0 \\ -x_5 + x_6 - 3x_7 &= 0 \\ 6x_5 + 3x_6 - 4x_7 &= 0 \end{aligned}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -3 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 3 & 0 \\ 0 & 0 & 0 & 0 & -6 & 3 & -4 & 0 \end{bmatrix}}_{-r_5\to r_5}$$

it is equivalent to

$$\begin{aligned} x_1 - x_4 &= 0 \\ x_2 - 2x_5 &= 0 \\ x_3 - x_6 &= 0 \\ x_4 - 3x_5 + x_6 - x_7 &= 0 \\ x_5 - x_6 + 3x_7 &= 0 \\ 6x_5 + 3x_6 - 4x_7 &= 0 \end{aligned}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -3 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & -14 & 0 \end{bmatrix}}_{6r_5+r_6\to r_6}$$

it is equivalent to

$$\begin{aligned} x_1 - x_4 &= 0 \\ x_2 - 2x_5 &= 0 \\ x_3 - x_6 &= 0 \\ x_4 - 3x_5 + x_6 - x_7 &= 0 \\ x_5 - x_6 + 3x_7 &= 0 \\ 3x_6 - 14x_7 &= 0 \end{aligned}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -3 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -\frac{14}{3} & 0 \end{bmatrix}}_{\frac{1}{3}r_6\to r_6}$$

it is equivalent to

$$\begin{aligned} x_1 - x_4 &= 0 \\ x_2 - 2x_5 &= 0 \\ x_3 - x_6 &= 0 \\ x_4 - 3x_5 + x_6 - x_7 &= 0 \\ x_5 - x_6 + 3x_7 &= 0 \\ x_6 - \tfrac{14}{3}x_7 &= 0 \end{aligned}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -14/3 & 0 \\ 0 & 0 & 0 & 1 & -3 & 0 & 11/3 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -5/3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -14/3 & 0 \end{bmatrix}}_{\substack{r_6+r_3\to r_3 \\ -r_6+r_4\to r_4 \\ r_6+r_5\to r_5}}$$

it is equivalent to

$$\begin{aligned} x_1 - x_4 &= 0 \\ x_2 - 2x_5 &= 0 \\ x_3 - \tfrac{14}{3}x_7 &= 0 \\ x_4 - 3x_5 - \tfrac{11}{3}x_7 &= 0 \\ x_5 - \tfrac{5}{3}x_7 &= 0 \\ x_6 - \tfrac{14}{3}x_7 &= 0 \end{aligned}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -10/3 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -14/3 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -4/3 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -5/3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -14/3 & 0 \end{bmatrix}}_{\substack{3r_5+r_4\to r_4 \\ 2r_5+r_2\to r_2}}$$

it is equivalent to

$$\begin{aligned} x_1 - x_4 &= 0 \\ x_2 - \tfrac{10}{3}x_7 &= 0 \\ x_3 - \tfrac{14}{3}x_7 &= 0 \\ x_4 - \tfrac{4}{3}x_7 &= 0 \\ x_5 - \tfrac{5}{3}x_7 &= 0 \\ x_6 - \tfrac{14}{3}x_7 &= 0 \end{aligned}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -4/3 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -10/3 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -14/3 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -4/3 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -5/3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -14/3 & 0 \end{bmatrix}}_{\substack{3r_5+r_4\to r_4 \\ 2r_5+r_2\to r_2}}$$

it is equivalent to

$$\begin{aligned} x_1 - \tfrac{4}{3}x_7 &= 0 \\ x_2 - \tfrac{10}{3}x_7 &= 0 \\ x_3 - \tfrac{14}{3}x_7 &= 0 \\ x_4 - \tfrac{4}{3}x_7 &= 0 \\ x_5 - \tfrac{5}{3}x_7 &= 0 \\ x_6 - \tfrac{14}{3}x_7 &= 0 \end{aligned}$$

Solving the system yields $x_1 = \frac{4}{3}x_7$, $x_2 = \frac{10}{3}x_7$, $x_3 = \frac{14}{3}x_7$, $x_4 = \frac{4}{3}x_7$, $x_5 = \frac{5}{3}x_7$ and $x_6 = \frac{14}{3}x_7$. Since x_7 a free variable we can be assigned any real number and we are dealing with elements or atoms or molecules, it is suitable to select values such that all the variables are become natural numbers. If we take $x_7 = 3$, we obtain $x_1 = x_4 = 4$, $x_2 = 10$, $x_3 = x_6 = 14$ and $x_5 = 5$. Therefore, the empirical formula for the reaction is

$$4MnSO_4 + 10NaBiO_3 + 14H_2SO_4 \longrightarrow 4Na_2MnO_4 + 5Bi_2(SO_4)_3 + 14H_2O + 3NaSO_4$$

IV. CONCLUSION

This study investigate every chemical reaction is represented by homogenous systems of linear equation only. Since homogeneous systems are always consistent (that is, it must have at least one solution), the chemical reaction is feasible if the homogeneous systems has many infinitely solutions. Thus, Gauss-Jordan elimination method is applicable for all possible cases in balancing chemical equations even for lower achieving students for success and it allows to become very fast and very accurate even with relatively difficult equations. Even if inspection is the usual method of balancing chemical equations, most chemistry students are confusing to use for an equation with many terms and the final result must be double checked to prove that the chemical equation is indeed balanced. Hence, the Gauss-Jordan method is systematic, can be applied to difficult reactions and can be easily used with equation solvers.

V. RECOMMENDATION

If the chemical reaction equation contain many numbers of elements in a reactants and products it is not easy to find the coefficients that appear in the equation by hand using Gauss-Jordan elimination method. Instead it is better to use the readers either MATLAB or C++ programs that have several advantages over other methods or language which is its basic data element is the matrix. Finally, I recommend that chemistry students should not be discouraged by past experiences in previous grades that convince them that they cannot do well in mathematics.

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