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A Standby server bulk arrival Queuing model of Compulsory server Vacation

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Abstract - We explore the steady state conduct of a $M^{[x]}/G/1$ queue with compulsory vacation. Here the arrival follows a poisson distribution. Service is rendered in two stages in which the second stage is optional. After the completion of service, the server has to undergo a compulsory vacation. During the time of vacation for the continuous service process, a very important concept of standby server is introduced. This provides a complete satisfaction towards the customers. We obtain in closed form, the steady state probability generating functions for the number of customers in the queue for various states of the server, the average number of customers as well as their average waiting time in the queue and the system.

Key words: optional second stage, compulsory vacation, standby server.

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1. Introduction

A few creators have examined queueing models in sorts of services in differing charges that incorporate stages of service and standby server. Assorted vacation approaches have been introduced by comprehended researchers in the before ponders. [1] Al-Jararha . J and Madan. K.C concentrated A M/G/1 line with second optional organization with general administration time dispersion. Chae.K.C et.al[2] inquired about M/G/1-sort lines with summed up get-aways. Choi. B.D.et.al.[3] made an examination on a M/G/1 line with various sorts of information, and gated get-aways. Cox. D.R[5] made an examination on Non-Markovian Stochastic Processes by the joining of Supplementary Variables. Cox. D.R and Miller, H .D. [6] concentrated the theory of Stochastic Processes. Madan and Anabosi [19] concentrated two sorts of administrations with single get-away and Bernoulli design outing. Maragathasundari[20] concentrated a mass arriving queueing model with three periods of administrations took after by benefit intrusion and postpone time. Choudhury[4] explored a bulb arrival queue with an excursion time under single vacatiom strategy. Madan and Abu-Dayyeh [15] concentrated a solitary server line with stage sort server departure and optional stage sort server vacation. Maraghi et.al [23] made an examination on the bunch landing queueing framework with second discretionary administration and irregular breakdown. Maragathasundari and Srinivasan [21] made an examination on a Non Markovian line with three phases of service and numerous vacations. Kavitha and Maragathasundari [13] researched the idea of limited tolerability and optional sorts of repair in a Non Markovian queue. A Non Markoviaan queue with discretionary services has been investigated by Srinivasan and Maragathasundari [26].A Non Markovian line with multi phases of service and reneging have been considered by Maragathasundari et.al[22]. Karthikeyan and Maragathasundari [12] made an examination on a bulk landing of two periods of administration with standby server in the midst of general get-away time and general repair time. Haridass and Arumuganathan[8] concentrated a retrial line in which modified outings under N course of action is solidified. Jain. M [9] investigated a working vacation queueing model with various sorts of server breakdowns. Time-subordinate properties of symmetric M/G/1 lines are considered by Kella.O et.al [10] Kumar.R and Sharma.S.K [11] reneging and Balking in a non markovian line. Madan. K.C. additionally, Chodhury. G[17] made an examination on M[x]/G/1 line with Bernoulli escape timetable under restricted admissibility.Madan. K.C [16], analyzed the line with two stage heterogeneous organization and binomial date-book server vacations..Maraghi. Et.al [24], concentrated a bunch Arrival fixing system with Random Breakdowns and Bernoulli Schedule server excursion taking after general conveyance. Ranjitham .A. furthermore, Maragathasundari .S[25] ,contemplated the two periods of administration in mass landing queueing model. In their audit, if an arriving gathering of customers find the server involved or in outing,, at that point the entire group joins the hover in order to search for the organization afresh. Ebrahim malalla and Madan K.C[7] considered a two periods of organizations, the central administration being optional with impedance and constrained openness of arrivals in the period of interruption in bunch section queueing models. Sowmiyah and Maragathasundari[27] investigated a mass queueing models with opened up journey and stages in repair. Kendall. D.G [14] concentrated the stochastic Processes occurring in the theory of lines and their examination by the system for embedded Markov chains.In view of all the above research work, another queueing model has been encompassed for the above flexible correspondence system. We infer that the Queueing model here proposed addresses a phase toward the headway of a general and symptomatic tractable showing instrument for the arrangement of compact correspondence sorts out under more sensible examinations.

2 The mathematical description of the model

The model is based on the following assumptions:

a) Customers' arrive one by one follows a compound Poisson process with a rate of arrival λ. Let λa_i dt(i = 1,2) be the first order probability that customers in batches of size i arrive at the system at a short interval of time (x. x+dt) , where $0\leq a_i \leq 1$. The concept of stand-by server starts to serve the customers when the original server is on vacation . We assume that the stand-by service time distribution follows an exponential distribution with stand-by service rate $\xi > 0$ and mean stand-by service time $1/\xi$.

b) The server provides service in two stages with optional second stage of service. As soon as the first stage of the server closes; the server has the choice of taking a optional second stage of service with the likelihood γ. The service follows general distributionThe service discipline is assumed to be on a first come first served basis (FCFS).Let us assume that the service time $\delta_i (i = 1,2)$ of the i th stage of service follows a general probability distribution with a distribution function $M_i(x)$ and probability density function $m_i(x)$. $i = 1,2$

Let $\delta_i(x) = \frac{m_i(x)}{1-M_i(x)}$ $i = 1,2$, $m_i(v) = \delta_i(v)e^{\left[-\int_0^v \delta_i(x)dx\right]}$, where $i = 1,2$. (a)

c) Once the service of a unit is finished, the server is accepted to take a compulsory vacation of general distribution. As soon as the first stage of the server closes and if no customer waits for optional second stage of service, then the server has the choice of taking a compulsory vacation with the likelihood β. Otherwise, after the completion of customers second stage of optional service, vacation will be followed. After the culmination of necessary vacation, it joins the system to proceed with a service to the holding up clients. Give us a chance to expect the Compulsory vacation time to be an irregular variable, after the general conditional probability law with distribution function U(x) and density function u(x). Here, let us consider that $\tau(x)$ is the conditional probability of a vacation period amid the interval (x,x+dx), given that the slipped by time is x, which can be given as

$\tau(x) = \frac{u(x)}{1-U(x)}$, $u(t) = \tau(t)e^{\left[-\int_0^t \tau(x)dx\right]}$ (b)

2.1. Notations

$Q_n^{(i)}(x,t)$: Probability that at time t,the server is active providing service and there are $n (n \geq 0)$ customers in the queue excluding the one being served in the i th stage of service and the elapsed service time for this customer is x.Consequently, $Q_n^{(i)}(t) = \int_0^\infty Q_n^{(i)}(x,t)\,dx$ denotes the probability that at time t there are n customers in the queue excluding the one customer in the i th optional stage of service irrespective of the value of x. $S_n(x,t)$:Probability that at time t, the server is on compulsory vacation with elapsed vacation time x and there are $n (n > 0)$ customers waiting in the queue for service. Consequently,$S_n(t) = \int_0^\infty S_n(x,t)\,dx$ denotes the probability that at time t there are n customers in the queue and the server is on compulsory vacation irrespective of the value of x. $R(t)$: Probability that at time$t$,there are no customers in the system and the server is idle but available in the system.

3. Steady state equation governing the system

$$\frac{d}{dx}Q_n^{(1)}(x,t) + (\lambda + \delta_1(x))Q_n^{(1)}(x) = \lambda \sum_{j=1}^{n-1} a_i\, Q_{n-i}^{(1)}(x) \quad ,n \geq 1 \qquad (3.1)$$

$$\frac{d}{dx}Q_n^{(1)}(x,t) + (\lambda + \delta_1(x))Q_n^{(1)}(x) = 0 \qquad (3.2)$$

$$\frac{d}{dx}Q_n^{(2)}(x,t) + (\lambda + \delta_2(x))Q_n^{(2)}(x) = \lambda \sum_{j=1}^{n-1} a_i\, Q_{n-i}^{(2)}(x) \quad ,n \geq 1 \qquad (3.3)$$

$$\frac{d}{dx}Q_n^{(2)}(x,t) + (\lambda + \delta_2(x))Q_n^{(2)}(x) = 0 \quad = 0 \qquad (3.4)$$

$$\frac{d}{dx}S_n(x) + (\lambda + \tau(x) + \xi)S_n(x) = \lambda \sum_{j=1}^{n-1} a_i\, S_{n-i}(x) + \xi S_{n+1}(x) \qquad (3.5)$$

$$\frac{d}{dx}S_0(x) + (\lambda + \tau(x) + \xi)S_0(x) = \xi S_1(x) \qquad (3.6)$$

$$\lambda R = \int_0^\infty S_0(x)\tau(x)dx \qquad (3.7)$$

Equations 3.1 – 3.7 are to be solved subject to the following boundary conditions:

$$Q_n^{(1)}(0) = \int_0^\infty S_{n+1}(x)\,\tau(x)dx + \lambda a_{n+1}R \quad ,n \geq 0$$

$$Q_n^{(2)}(0) = \gamma \int_0^\infty Q_n^{(1)}(x)\delta_1(x)dx \qquad n \geq 0$$

$$S_n(0) = \beta \int_0^\infty Q_n^{(1)}(x)\delta_1(x)dx + \int_0^\infty Q_n^{(2)}(x)\delta_2(x)dx \qquad (3.8)$$

3.1 Queue size Distribution at Random Epoch

We define the probability generation function as follows:

$$Q_q^{(i)}(x,z) = \sum_{n=0}^\infty z^n\, Q_q^{(i)}(x), \qquad Q_q^{(i)}(z,t) = \sum_{n=0}^\infty z^n\, Q_q^{(i)} \qquad i = 1,2$$

$$S_q(x,z) = \sum_{n=0}^\infty z^n\, S_n(x,t) \qquad (A)$$

$$S_q(z,t) = \sum_{n=0}^\infty z^n S_n$$

$$a(z) = \sum_{n=1}^\infty a_i z^i\,; \quad |z| \leq 1$$

4. Steady state queue size distribution at a random epoch

Cox (1955) has investigated non-Markovian models by changing them into Markovian ones, through the presentation of at least one supplementary factors. A stable recursive plan for the estimation of the restricting probabilities can be created, in view of an adaptable regenerative approach

Multiplying eq. 3.1 by z^n ,summing over n and adding the result to eq. (3.2) and again using (A), we get

$$\frac{d}{dx}Q_q^{(1)}(x,z) + (\lambda + \delta_1(x) - \lambda a(z))Q_q^{(1)}(x)Q^{(1)}(x,z) = 0 \qquad (4.1)$$

Similarly,

$$\frac{d}{dx}Q_q^{(2)}(x,z) + (\lambda + \delta_2(x) - \lambda a(z))Q_q^{(2)}(x)Q^{(2)}(x,z) = 0 \qquad (4.2)$$

$$\frac{d}{dx}S_q(x,z) + \left(\lambda + \tau(x) + \xi - \frac{\xi}{z} - \lambda a(z)\right)S_q(x,z) = 0 \qquad (4.3)$$

Next, similar operations are carried out on the boundary conditions 3.8, we getType equation here.

$$zQ_q^{(1)}(0,z) = \int_0^\infty S_q(x,z)\,\tau(x)dx + \lambda(a(z)-1)R \qquad (4.4)$$

$$Q_q^{(2)}(0,z) = \gamma\int_0^\infty Q_q^{(1)}(x,z)\delta_1(x)dx \qquad (4.5)$$

$$S_q(0,z) = \beta\int_0^\infty Q_q^{(1)}(x,z)\delta_1(x)dx + \int_0^\infty Q_q^{(2)}(x,z)\delta_2(x)dx \qquad (4.6)$$

Now we integrate equations 4.1 – 4.3 between the limits 0 and x.

$$Q_q^{(1)}(x,z) = Q_q^{(1)}(0,z)\,e^{-(\lambda a(z)-\lambda)x-\int_0^x \delta_1(t)dt} \qquad (4.7)$$

$$Q_q^{(2)}(x,z) = Q_q^{(2)}(0,z)\,e^{-(\lambda a(z)-\lambda)x-\int_0^x \delta_2(t)dt} \qquad (4.8)$$

$$S_q(x,z) = S_q(0,z)\,e^{-\omega x-\int_0^x \tau(t)dt} \qquad (4.9)$$

Where $\omega = (\lambda + \xi - \frac{\xi}{z} - \lambda a(z))$

$Q_q^{(1)}(0,z)$, $Q_q^{(2)}(0,z)$, $S_q(0,z)$ are given in equations (4.4) - (4.6)

Next we integrate 4.7 – 4.9 with respect to x, by parts, we get

$$Q_q^{(1)}(z) = Q_q^{(1)}(0,z)\frac{1-\bar{M}_1(\lambda-\lambda a(z))}{\lambda-\lambda a(z)} \qquad (4.10)$$

$$Q_q^{(2)}(z) = Q_q^{(2)}(0,z)\frac{1-\bar{M}_2(\lambda-\lambda a(z))}{\lambda-\lambda a(z)} \qquad (4.11)$$

$$S_q(z) = S_q(0,z)\frac{1-\bar{U}(\omega)}{\omega} \qquad (4.12)$$

Where $\bar{M}_i(\lambda-\lambda a(z)) = \int_0^\infty e^{-(\lambda a(z)-\lambda)x}\, dM_i(x)$, i= 1,2 is the Laplace Stieltjes transform of the ith service time and $\bar{U}(\omega) = \int_0^\infty e^{-(\omega)x}\, dU(x)$ is the Laplace Stieltjes transform of the compulsory vacation.

To find $\int_0^\infty Q_q^{(1)}(x,z)\delta_1(x)dx$, $\int_0^\infty Q_q^{(2)}(x,z)\delta_2(x)dx$ and $\int_0^\infty S_q(x,z)\,\tau(x)dx$

For this purpose, we multiply the equations 4.7 – 4.9 by $\delta_1(x), \delta_2(x)$ and $\tau(x)$ respectively and integrate each with respect to x.

Hence we get $\int_0^\infty Q_q^{(1)}(x,z)\delta_1(x)dx = Q_q^{(1)}(0,z)\bar{M}_1(\lambda-\lambda a(z))$ (4.13)

$$\int_0^\infty Q_q^{(2)}(x,z)\delta_2(x)dx = Q_q^{(2)}(0,z)\bar{M}_2(\lambda-\lambda a(z)) \qquad (4.14)$$

$$\int_0^\infty S_q(x,z)\,\tau(x)dx = S_q(0,z)\,\bar{U}(\omega) \qquad (4.15)$$

Using equations 4.13 -4.15 into equations 4.4 – 4.6 and further applying the equations 4.10 – 4.12, we get

$$Q_q^{(1)}(z) = \frac{-R(1-\bar{M}_1(\lambda-\lambda a(z))}{z-\beta\bar{M}_1(\lambda-\lambda a(z))\bar{U}(\omega)-\gamma\bar{M}_1(\lambda-\lambda a(z))\bar{M}_2(\lambda-\lambda a(z))\bar{U}(\omega)} \qquad (4.16)$$

$$Q_q^{(2)}(z) = \frac{-R\gamma\,(1-\bar{M}_2(\lambda-\lambda a(z))\bar{M}_1(\lambda-\lambda a(z))}{z-\beta\bar{M}_1(\lambda-\lambda a(z))\bar{U}(\omega)-\gamma\bar{M}_1(\lambda-\lambda a(z))\bar{M}_2(\lambda-\lambda a(z))\bar{U}(\omega)} \qquad (4.17)$$

$$S_q(z) = \frac{-R\left(1-\bar{U}(\omega)\right)(\lambda-\lambda a(z))(\beta\bar{M}_1(\lambda-\lambda a(z))+\gamma\bar{M}_1(\lambda-\lambda a(z))\bar{M}_2(\lambda-\lambda a(z))}{z-\beta\bar{M}_1(\lambda-\lambda a(z))\bar{U}(\omega)-\gamma\bar{M}_1(\lambda-\lambda a(z))\bar{M}_2(\lambda-\lambda a(z))\bar{U}(\omega)} \qquad (4.18)$$

Let $D_q(z) = Q_q^{(1)}(z) + Q_q^{(2)}(z) + S_q(z)$ be the probability generating function of the queue size.

To determine the idle time R

Using the normalizing condition R + $D_q(1)$ = 1, we get R. (4.19)

For this purpose we apply the following steps:

Eq.3.27 is indeterminate of the form $\frac{0}{0}$ at z = 1.Hence L Hopital's rule is applied. As a result we get $D_q(1) = \frac{(-\lambda E(I)+\xi)(\lambda E(I)\gamma E(M_2))+(-\lambda E(I)+\delta)^2\ (\lambda E(I)E(M_1)+\gamma E(M_2))+2\lambda E(I)E(v)(-\lambda E(I)+\delta)(\beta+\gamma)}{(-\lambda E(I(I-1)-\xi)(1-(\beta+\gamma))+2(-\lambda E(I)+\delta)^2\ (1-(\beta+\gamma))}$

(4.20)

Where E(I) is the mean size of the bulk arrival, $E(M_i)$ is the mean service time, i = 1,2 and E(U) is the mean vacation time.

Applying (3.29) in (3.28), we get

$$R = \frac{(-\lambda E(I(I-1)-\xi)(1-(\beta+\gamma))+2(-\lambda E(I)+\xi)^2\ (1-(\beta+\gamma))}{((-\lambda E(I(I-1)-\xi)(1-(\beta+\gamma))+2(-\lambda E(I)+\delta)^2\ (1-(\beta+\gamma))+ (-\lambda E(I)+\xi)(\lambda E(I)\gamma E(M_2))+(-\lambda E(I)+\xi)^2\ (\lambda E(I)E(M_1)+\gamma E(M_2))+2\lambda E(I)E(v)(-\lambda E(I)+\delta)(\beta+\gamma)} \qquad (4.21)$$

Also we obtain the utilization factor ρ using the relation ρ = 1- R i.e $\rho = \frac{(-\lambda\, E(I)+\delta)(\lambda\, E(I)\gamma E(M_2))+(-\lambda\, E(I)+\delta)^2\ (\lambda\, E(I)E(M_1)+\gamma E(M_2))+2\lambda\, E(I)E(v)(-\lambda\, E(I)+\delta)(\beta+\gamma)}{((-\lambda\, E(I(I-1)-\xi)(1-(\beta+\gamma))+2(-\lambda\, E(I)+\delta)^2\ (1-(\beta+\gamma))+ (-\lambda\, E(I)+\delta)(\lambda\, E(I)\gamma E(M_2))+(-\lambda\, E(I)+\delta)^2\ (\lambda\, E(I)E(M_1)+\gamma E(M_2))+2\lambda\, E(I)E(v)(-\lambda\, E(I)+\delta)(\beta+\gamma)}$ (4.22)

5. Steady state mean queue size at a random epoch

Let L_q denote the steady state mean queue size at a random epoch.

Then using the Tauberian property,we have $L_q = \frac{d}{dq} D_q(z)_{z=1}$

Since $D_q(z)$ is indeterminate of the form $\frac{0}{0}$ at z = 1, we apply the following formula:

$$L_q = \lim_{z\to 1} \frac{D''N''' - N''D'''}{2(D'')^2} \quad (5.1)$$

Where double and triple primes denote the second and third order derivatives as follows:

$D'' = (-\lambda\, E(I(I-1)-\xi)(1-(\beta+\gamma)) + 2(-\lambda\, E(I)+\delta)^2\ (1-(\beta+\gamma))$

$N''' = (-\lambda\, E(I)+\xi)\ (\lambda\, E(I))(E(M_1)+2\gamma E(M_2))$

$N'' = (-\lambda\, E(I)+\xi)(\lambda\, E(I)\gamma E(M_2)) + (-\lambda\, E(I)+\xi)^2\ (\lambda\, E(I)E(M_1)+\gamma E(M_2)) + 2\lambda\, E(I)E(v)(-\lambda\, E(I)+\delta)(\beta+\gamma)$

$D''' = -3\,(-\lambda\, E(I)+\delta)^2\ (\beta\lambda\, E(I)E(M_1)+\lambda\, E(I)\gamma(E(M_2)+E(M_1))) - 2(-\lambda\, E(I(I-1)-2\xi)\ \ (-\lambda\, E(I)+\delta)(\beta+\gamma) +$ $(1-(-\lambda\, E(I(I-1)-\xi)\ (-\lambda\, E(I)+\delta)(\beta+\gamma))$ (5.2)

Substituting (5.2) in (5.1) , we obtain L_q in a closed form. Further using L_q in Little's formula we obtain the other required performance measures as follows:

Average number of customers in the system $L = L_q + \rho$ where ρ is given by eq.4.22

Average waiting time in the queue $W_q = \frac{L_q}{\lambda}$

Average waiting time in the system $W = \frac{L}{\lambda}$

6. Conclusion

The perspective of discretionary stage of administration has been displayed in this model since it set up the authentic conditions about. The peculiarity of this work is the introduction of important concept, optional stage of administration in the knowledge which gives complete satisfaction to all the arriving customers. The imperative idea of standby server has been plainly all around characterized in this model which assumes an exceptionally conspicuous part in the greater part of the lining framework categories. This display has dormant helpful solid life application in media transmission structure, therapeutic interpretation and collecting organizations. As a pending work, additional components of Priority organization set up time and close down time can be fused and a comparable examination of execution strategies with this model can be figured. Additional piece of retrial line can moreover be considered.

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