Estimation of open channel flow parameters by using optimization techniques

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ABSTRACT: Open channel flow parameter estimation is an inverse problem, which involves the prediction of a function within a domain, given an error criterion with respect to a set of observed data. Various numerical methods have been developed to estimate open channel flow parameters. For this study, Genetic Algorithm optimization technique is selected. Because of its inherent characteristics, Genetic Algorithm optimization technique avoids the subjectivity, long computation time and ill-posedness often associated with conventional optimization techniques.

The present study involves estimation of open channel flow parameters having different bed materials invoking data of Gradual Varied Flow (GVF). Use of GVF data facilitates estimation of flow parameters. The necessary data base was generated by conducting laboratory experiments in Hydraulics Lab of civil Engineering at IIT Roorkee. In the present study, the efficacy of the Genetic Algorithm (GA) optimization technique is assessed in estimation of open channel flow parameters from the collected experimental data. Computer codes are developed to obtain optimal flow parameters Optimization Technique. Applicability, Adequacy & robustness of the developed code are tested using sets of theoretical data generated by experimental work. Estimation of Manning’s Roughness coefficient from the collected experimental work data by using Manning’s equation & GVF equation were made.

The model is designed to arrive at such values of the decision variables that permit minimized mismatch between the observed & the computed GVF profiles. A simulation model was developed to compute GVF depths at preselected discrete sections for given downstream head and discharge rate. This model is linked to an optimizer to estimate optimal value of decision variables. The proposed model is employed to a set of laboratory data for three bed materials (i.e, d_{50}=20mm, d_{50}=6mm and lined concrete). Application of proposed model reveals that optimal value of fitting parameter ranges from 1.42 to 1.48 as the material gets finer. This value differs from the currently documented value i.e. 1.5. The optimal estimates of Manning’s n of three
different bed conditions of experimental channel appear to be higher than the corresponding reported \textit{Strickler’s} estimates.

\textbf{Key Words:-} Open channel flow parameters, GVF, parameter estimation, optimization techniques, Manning’s roughness coefficient.

\section*{INTRODUCTION}

Parameter identification techniques have been widely used in the field of hydrology, meteorology, and oceanography. The issue of parameter identification based on the optimal control theories in oceanography can be traced from the early work of Bennett and McIntosh (1982) and Prevost and Salmon (1986). Panchang and O’Brien (1989) carried out early an adjoint parameter identification for bottom drag coefficient in a tidal channel. Das and Lardner (1991) estimated the bottom friction and water depth in a two-dimensional tidal flow. Yeh and Sun (1990) presented an adjoint sensitivity analysis for a groundwater system and identified the parameters in a leaky aquifer system. Wasantha Lal (1995) used singular value decomposition to calibrate the Manning’s roughness in one-dimensional (1D) Saint Venant equations. Khatibi et al. (1997) identified the friction parameter in 1D open channel considering the selection of performance function and effect of uncertainty in observed data. Atanov et al. (1999) Used the adjoint equation method to identify a profile of Manning’s $n$ in an idealized trapezoidal open channel. Ishii (2000) identified a constant Manning’s $n$ in an open channel flow with a movable bed. Ramesh et al. (2000) solved the inverse problem of identifying the roughness coefficient in a channel network using the sequential quadratic programming algorithm. Sulzer et al. (2002) estimated flood discharges using the Levenberg–Marquardt minimization algorithm. For the parameter identification issues about adjoint methodology in meteorology and oceanography, one may refer to Ghil and Malanotte-Rizzoli (1991) and Zou et al. (1992).

The identifications of parameters in some cases are hard to achieve due to ill-posedness in the inverse problems. Chavent (1974) noted instability and nonuniqueness of identified parameters in the distributed system. Due to the instability, some minimization procedures will lead to serious errors in the identified parameters and make the identification process unstable. In the case of nonuniqueness, the identified parameters will differ according to the initial estimations of the parameters, and not converge to their optimal (or “true”) values. Yeh (1986) and Navon
(1998) have pointed out that the problem of uniqueness in parameter identification is intimately related to identification, which addresses the question of whether it is at all possible to obtain a unique solution of the inverse problem for unknown parameters. Although there are a lot of identification procedures available for estimating parameters in mathematical models, none of them can automatically guarantee stability and uniqueness in the parameter identifications in diverse engineering problems. It is therefore vital to confirm the performance of these procedures to find stable ones that can warrant obtaining the optimal solutions. For the present study, channel roughness is identified by using optimization technique.

Optimization techniques were successfully used by Becker and Yeh (1972, 1972a), Fread and Smith (1978) and Wormleaton and Karmegam (1984) to identify parameters for regular prismatic channels having simple cross-sections. These researchers used the same optimization algorithm (the so-called "Influence Coefficient" Algorithm) which, mathematically, is closely related to both quasi linearization and the gradient method. Khatibi et al. (1997) used a nonlinear least square technique with three types of objective function and identified open channel friction parameters by a modified Gauss-Newton method. Atanov et al. (1999) used Lagrangian multipliers and a least square errors criterion to estimate roughness coefficients. More recently, Ding et al. (2004) used the quasi-Newton method to identify Manning’s roughness coefficients in shallow water flows. Nevertheless, the above studies considered only the case of in-bank flow. Therefore, there is a need to extend the method to out-bank flow, where flood plain roughness will obviously have to be considered.

One of the very few studies which dealt with the identification of compound channel flow parameters is the one by Nguyen and Fenton (2005). In this study, roughness coefficients in the main channel and flood plains were identified as two different parameters using an automatic optimization method. The model was applied to Duong River in Vietnam, where roughness coefficients of the main channel and the flood plain were presented as different constant values as well as polynomial functions of stage.

**Need for the Present Study:** From above brief literature review it can be seen that many investigators made many experimental study on identification of open channel flow parameters
by using optimization techniques. Still more experimental study is required to estimate open channel flow parameters. The present study investigated for estimation of channel roughness coefficients for different three types of bed materials ($d_{50}=20\text{mm}$, $d_{50}=6\text{mm}$ and lined concrete) by using optimization method. Also the present study is done to generate and monitor gradually varied flow profiles corresponding to different bed materials, discharge rates and ponded depths in the channel by using Crank-Nicolson method to solve the governing differential equation.

In this study the Simplex method is used in an optimization model to estimate the parameters in the channel.

**Objectives:** The present study involves estimation of Manning roughness $n$ of a channel having different channel bed materials invoking data of gradual varied flow (GVF). Use of GVF data facilitates estimation of depth dependent Manning’s roughness $n$. the necessary data base was generated by conducting laboratory experiments. The overlying objective is fulfilled through the accomplishment of sub objectives listed below. To identify open channel flow parameters by using Genetic Algorithm optimization Technique, To generate and monitor gradually varied flow profiles corresponding to different bed materials, discharge and ponded depths and Invoking the observed data of the GVF profiles and the linked simulation optimization approach to estimates Manning’s $n$ corresponding to different channel bed materials in the experimental channel, and hence to calibrate the following composite roughness equation.

**METHODS AND MATERIALS**

This study was carried out to identify open channel flow parameters by using Genetic Algorithm optimization technique. Manning’s roughness coefficient and other parameters are estimated for different bed materials used ($d_{50}=6\text{mm}$ and $20\text{mm}$ grain size and Lined concrete bed materials). Also, GVF flow profile is identified. Crank-Nicolson method is used to solve the governing differential equation.

Parameter optimization technique is used to find the optimal value of coefficient roughness for three different bed materials. Estimation of roughness coefficient is based on Manning’s equation for estimation of manning roughness coefficient and corresponding manning roughness
parameters. This estimation invokes the data of observed GVF profiles and such accounts for different bed materials with the flow depth. Experimental works was done to several sets of data monitored in Hydraulics Laboratory of Civil Engineering Department.

**Experimental Works**

In this chapter, water surface flow profiles corresponding to specific discharges, bed material and ponded depth have been obtained through experimentation. This chapter includes a detailed description of experimental setup, adopted procedures and the observations with range of data obtained for different flow conditions. The experiments for the investigation were carried out in Hydraulics Laboratory of Civil Engineering Department. IIT-Roorkee, India

**Details of Experimental Setup**

**Flume**

A rectangular tilting flume of length 30m, width 0.205m and height 0.50m was used. The bed of the flume was made up of lined concrete and the other two sides were made up of glass and GI sheet. Discharge was released through an inlet pipe of 0.010m diameter into the flume. The entrance of the channel was provided with flow suppressors in order to make the flow stable. In order to maintain desired depth of water at the downstream of the channel, a tail gate was fitted at the end of the channel. Water discharging from the tail gate, passed to the sump which was circulated again through a 15hp centrifugal pump for further experimentation.

**Experimental Procedures**

The experiments were conducting by adopting the following steps as mentioned below:-

**Slope Measurement**

All the sets of experiment were performed on a particular slope of the channel. The slope was measured by using two steel containers connected with a long rubber tube. Both the containers were placed on the channel bed separated by the rubber tube along the length of the channel. One of the containers placed at higher elevation was filled with water and simultaneously care was taken to remove air bubble from the connecting tube. They are left undisturbed for sufficient
amount of time around 24 hours. Then the water levels were measured. The slope of the channel was computed by using the following formula.

\[ S_o = \frac{H_1 - H_2}{L} \]  

(1)

Where, \( H_2 \) and \( H_1 \) is the depth of water in second and first container respectively after equilibrium is established and \( L \) is the distance between the containers.

Based on this formula and obtained data after 24 hours, the bed slope of the channel will be:

\[ H_1 = 21.5 \text{cm} = 0.215 \text{m}, \quad H_2 = 7.792 \text{cm} = 0.0792 \text{m} \]

\[ L = 22.7 \text{m} \]

Then, \[ S_o = \frac{H_1 - H_2}{L}, \quad S_o = \frac{0.215 - 0.0792}{22.7}, \quad S_o = 0.00598 \]

Therefore, the bed slope of the channel is 0.00598.

**Sieve Analysis**

Sieve analysis was performed to determine the particle size of the material used to create artificial bed roughness. Results of sieve analysis were plotted to investigate the particle size of the bed material used in the present study. Experiments were conducted on two different bed materials. First on one rough bed condition having gravel as a bed particle size \( d_{50} = 20 \text{mm} \), \( d_{50} = 6 \text{mm} \) and then on the smooth condition having lined concrete as bed material. Then, the gradation curve is plotted as follow:

![Gradation curve](image-url)
Calibration of orifice meter

Orifice meter was provided in the inlet pipe for the measurement of discharge. Orifice plate was made up of GI sheet having diameter of 0.06m and the diameter of inlet pipe was 0.10m. Ultrasonic flow meter was used for the calibration of coefficient of discharge of orifice meter. Different discharges were noted corresponding to varying head. This result was plotted and the best fitted line was used (Fig. 3).
Fig. 3 Calibration curve

Cd was calibrated as 0.66. after calibration of Cd of orifice meter, the discharge in the channel was computed by using the following equation.

\[ Q = C_d a_o \sqrt{2gh} \]  

(2)

Where, \( a_o \) is area of orifice plate; \( g \) = acceleration due to gravity and \( h \) = height of water column.

Measurement of water surface profiles

i) Water was released into the rectangular flume by opening the valve of inlet pipe,

ii) The desired depth of flow was maintained at the downstream end by operating sluice gate provided at the end of the channel. The depth of water was measured using pointer gauge,

iii) After a while when the flow become steady in the channel and the desired depth was maintained at the downstream end, the water surface profile was being measured,

iv) Starting from the maintained depth at the downstream end (0.00m), the water surface profile is measured towards upstream at 21 discrete locations that are 0.00m, 0.20m, 0.70m, 1.20m, 1.70m, 2.20m, 2.70m, 3.70m, 4.70m, 5.70m, 6.70m, 7.70m, 8.70m, 9.70m, 10.70m, 12.70m, 14.70m, 16.70m, 18.70m, 20.70m and 22.70m.

v) The above mentioned steps were repeated for three different downstream depths, Discharges rates and bed roughness as mentioned in Table 1

Table 1 Data used for experimental measurement of water surface profiles

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<th>Discharge rates (m³/s)</th>
<th>8.601x10⁻³</th>
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<th>9.314x10⁻³</th>
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<td>Bed materials (d₅₀ in mm)</td>
<td>d₅₀=20</td>
<td>d₅₀=6</td>
<td>Lined concrete</td>
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</table>
Collection of data

The data obtained for experimental measured water surface profiles corresponding to different bed materials is presented in Table 2, 3 and 4 for $d_{50}=20\text{mm}$, $d_{50}=6\text{mm}$ and lined concrete respectively.

Table 2 Observed water surface profiles corresponding to $d_{50}=20\text{mm}$

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<th>y21(m)</th>
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<td>0.2163</td>
<td>0.2588</td>
<td>0.307</td>
<td>0.2155</td>
<td>0.2608</td>
<td>0.3061</td>
</tr>
<tr>
<td>13</td>
<td>8.7</td>
<td>0.2027</td>
<td>0.2526</td>
<td>0.3027</td>
<td>0.2074</td>
<td>0.2498</td>
<td>0.2997</td>
<td>0.21</td>
<td>0.2509</td>
<td>0.3017</td>
</tr>
<tr>
<td>14</td>
<td>9.7</td>
<td>0.1992</td>
<td>0.2491</td>
<td>0.2973</td>
<td>0.2011</td>
<td>0.2507</td>
<td>0.2944</td>
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<tr>
<td>15</td>
<td>10.7</td>
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<td>0.2419</td>
<td>0.2892</td>
<td>0.1898</td>
<td>0.2363</td>
<td>0.2844</td>
<td>0.1955</td>
<td>0.2437</td>
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<td>16</td>
<td>12.7</td>
<td>0.1745</td>
<td>0.2263</td>
<td>0.2727</td>
<td>0.1755</td>
<td>0.2206</td>
<td>0.2678</td>
<td>0.1781</td>
<td>0.2245</td>
<td>0.2717</td>
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<tr>
<td>17</td>
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<td>0.1662</td>
<td>0.2144</td>
<td>0.2627</td>
<td>0.1673</td>
<td>0.2126</td>
<td>0.2588</td>
<td>0.1681</td>
<td>0.2145</td>
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<tr>
<td>18</td>
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<td>0.1509</td>
<td>0.2016</td>
<td>0.25</td>
<td>0.1519</td>
<td>0.1989</td>
<td>0.2434</td>
<td>0.1563</td>
<td>0.2018</td>
<td>0.2508</td>
</tr>
<tr>
<td>19</td>
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<td>0.1934</td>
<td>0.2418</td>
<td>0.1428</td>
<td>0.187</td>
<td>0.2323</td>
<td>0.1456</td>
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<td>0.2344</td>
</tr>
<tr>
<td>20</td>
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<td>0.1428</td>
<td>0.1909</td>
<td>0.2383</td>
<td>0.1358</td>
<td>0.1835</td>
<td>0.2279</td>
<td>0.1365</td>
<td>0.1813</td>
<td>0.2309</td>
</tr>
<tr>
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<td>22.7</td>
<td>0.1327</td>
<td>0.1803</td>
<td>0.2276</td>
<td>0.1238</td>
<td>0.1688</td>
<td>0.2179</td>
<td>0.1273</td>
<td>0.1736</td>
<td>0.2226</td>
</tr>
</tbody>
</table>

Table 3 Observed water surface profiles corresponding to d₅₀=6mm

Table 4 Observed water surface profiles corresponding to lined concrete
RESULTS AND DISCUSSION

Simulation Model

The optimization problem posed in the preceding section is solved by employing the linked optimization problem. This approach would require development of a model for simulation of GVF depths at preselected discrete sections for given downstream head and discharge rate.
Subsequently this simulation model is linked to an optimizer for addressing the optimization problem. Effectively the simulation model would provide the vector of computed depths \( y(x_i, Q_k, H_l) \) appearing in the objective function. The details of the simulation model in the following sections.

**Discretization of reach**

In the simulation model the entire channel reach is discretized into \( M \) small space steps such that depth of water level at \( M^{th} \) step is greater than 1.01 x normal depth.

**Governing differential equation**

Governing differential equation used for simulation of GVF is given as:

\[
\frac{dy}{dx} = \frac{S_o - S_f}{1 - \frac{Q^2 T}{g A^3}}
\]  

(3)

In this equation \( \frac{dy}{dx} \) is change in depth \( y \) with distance \( x \); \( S_f \) is energy slope and \( T \) is top width. \( S_f \) can be calculated by using Manning’s formula as:

\[
S_f = \frac{n_c^2 Q^2}{A^2 R^{5/3}}
\]  

(4)

Where \( n_c \) is composite roughness coefficient and computed as follow:

\[
n_c = \left( \frac{\sum_{i=1}^{N} n_i^x P_i}{\left( \sum_{i=1}^{N} P_i \right)^{1/x}} \right)^{1/x}
\]  

(5)

**Simulation strategy**

Crank-Nicolson method is used to solve the governing differential equation mentioned in above section. In this method, depth of water level at next space step is calculated as:

\[
y_{i+1} = y_i - \beta \Delta x
\]  

(6)
Where, $y_{i+1}$ and $y_i$ is depth of water level at $i + 1^{th}$ and $i^{th}$ section respectively, $\Delta x$ is the distance between them and $\beta$ is the average slope which is given as follow:

$$
\beta = \frac{\left(\frac{dy}{dx} \bigg| y_i + \frac{dy}{dx} \bigg| y_{i+1}\right)}{2}
$$

(7)

Where, $(\frac{dy}{dx} \bigg| y_i)$ and $(\frac{dy}{dx} \bigg| y_{i+1})$ are the change in the depth of flow with channel distance $x$ at $i^{th}$ and $i + 1^{th}$ section. Equation (6) can be further elaborated using previously mentioned equation as:

$$
y_{i+1} = y_i - \frac{\left(S_o - \frac{n^2_{cl}Q^2}{T^2y_i^{10/3}}S_o - \frac{n^2_{cl}Q^2}{1 - \frac{Q^2}{gT^2y_i^3}}\right)\Delta x}{\frac{Q^2}{1 - \frac{Q^2}{gT^2y_{i+1}^3}}}
$$

(8)

Where, $n_{cl}$ and $n_{cl+1}$ are the composite roughness of $i^{th}$ and $i + 1^{th}$ section. An iterative procedure is adopted for the computation of $y_{i+1}$. In this procedure, $y_{i+1}^{l+1}$ is calculated where $l$ is the number of iteration as:

$$
y_{i+1}^{l+1} = y_i - \beta \Delta x
$$

(9)

And the iteration ends when it met the converging criterion, which is given as:

$$
\left|y_{i+1}^{l+1} - y_{i+1}^l\right| < \varepsilon
$$

(10)

Where, $\varepsilon$ is a constant term. Thus, using the above mentioned approach $y_{i+1}$ is computed for each discrete step up to $M^{th}$ step and this leads to the simulation of GVF profiles.

**Simulator**
As mentioned in above section, the experimental channel consists of three types of wetted perimeter; accordingly following equation is used in the simulator for computing the composite roughness $n_c$:

$$n_c = \left( n_1^\varepsilon B + n_2^\varepsilon y + n_3^\varepsilon y \right)^{1/\varepsilon} \left( B + 2y \right)^{1/\varepsilon}$$  (11)

Fig. 4 Composite roughness of channel

Where, $n_c$ is the composite Manning’s $n$, $n_1$, $n_2$, and $n_3$ are value of Manning’s $n$ for bed and sides respectively. $B$ is bed width and $y$ is the depth of flow. Since composite roughness depends on the depth of the flow, which is not constant in the present scenario. Therefore, $n_c$ is computed at each section of the water surface flow profile. The value of $\varepsilon$ is take $n$ as 0.001 in equation (5).

**Optimization**

The following problem was solved three times corresponding to different bed conditions i.e. $d_{50}=20$mm, $d_{50}=6$mm and lined concrete as bed materials.
Decision Variables:

\( (n_i, i = 1, \ldots, 3) \); and \( \propto \)

Objective Function:

\[
\text{Min } Z = \sum_{i}^{3} \sum_{k}^{3} \sum_{l}^{M} w_i [y(x_i, Q_k, H_l) - \widehat{y}_{ikl}]^2
\]  

(12)

Where, \( y(x_i, Q_k, H_l) \) and \( \widehat{y}_{ikl} \) are simulated and experimentally measured depth at \( i^{th} \) discrete section, \( k^{th} \) discharge rate and \( l^{th} \) downstream head respectively; \( M \) is a subset of the locations where the observed depth is larger than 1.01 x normal depth; \( w_i \) is the weight assigned to the mismatch at \( i^{th} \) location. In the present study the weights are assigned to index the length discretized by the discrete sections. Thus \( (w_i) \) is defined as follows:

\[
w_i = \frac{(x_{i+1} - x_{i-1})}{2}
\]  

(13)

Constraint:

i) Following six constraints were assigned to impose upper and lower limits of the segment roughness coefficients ( \( n_{\text{max}} \) and \( n_{\text{min}} \), \( i = 1, \ldots, 3 \)).

\[
n_{\text{max}} \geq n_{\text{min}} , i = 1, \ldots, 3
\]  

(14)

The adopted values of the limits are given in Table 5.

Table 5 Upper and lower limits of roughness coefficients

<table>
<thead>
<tr>
<th></th>
<th>( n_1 )</th>
<th>( n_2 )</th>
<th>( n_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_{\text{max}} )</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>( n_{\text{min}} )</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
</tbody>
</table>
ii) Following three constraints were assigned to ensure realistic relative roughness of the three roughness coefficients.

\[ n_1 \geq n_2 \geq n_3 \]  

(15)

iii) Following constraints was assigned to impose upper and limits of fitting parameters (\( \propto \)).

\[ 2 \geq \propto \geq 1 \]  

(16)

Since the reported value of \( \propto \) 1.5, a range of 1 to 2 was prescribed.

Linked simulation optimization approach is used to estimate the optimal values of the parameters for three bed conditions i.e \( d_{50}=20 \text{mm} \), \( d_{50}=6 \text{mm} \) and lined concrete as bed materials and their corresponding GVF profiles were simulated.  

**Optimal values**

Optimal values of decision variables and their corresponding minimized objective function value for different bed materials are mentioned in Table 6.

Table 6 Optimal values of decision variables and objective function.

<table>
<thead>
<tr>
<th>Bed materials</th>
<th>( n_1 )</th>
<th>( n_2 )</th>
<th>( n_3 )</th>
<th>( \propto )</th>
<th>( Min Z (m^2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_{50}=20 \text{mm} )</td>
<td>0.034</td>
<td>0.016</td>
<td>0.018</td>
<td>1.42</td>
<td>1.16x10^-4</td>
</tr>
<tr>
<td>( d_{50}=6 \text{mm} )</td>
<td>0.030</td>
<td>0.016</td>
<td>0.018</td>
<td>1.46</td>
<td>1.62x10^-4</td>
</tr>
<tr>
<td>Lined concrete</td>
<td>0.027</td>
<td>0.015</td>
<td>0.017</td>
<td>1.48</td>
<td>1.09x10^-4</td>
</tr>
</tbody>
</table>

**Optimal reproduction of GVF profiles**

Computed GVF profiles corresponding to the optimal parameter values and the variation of composite roughness are in the following figures. The profile is plotted for three different bed materials corresponding to discharge rates and water depth.
Fig. 5 Observed reproduction of GVF profiles ($Q=8.601\times10^{-3}$ m$^3$/s and $d_{50}=20$ mm)

Fig. 6 Optimal reproduction of GVF profiles ($Q=8.601\times10^{-3}$ m$^3$/s and $d_{50}=20$ mm)
Fig. 7 Observed reproduction of GVF profiles (Q=9.233x10^{-3} m^3/s and d_{50}=6mm)

Fig. 8 Optimal reproduction of GVF profiles (Q=9.233x10^{-3} m^3/s and d_{50}=6mm)
Fig. 9 Observed reproduction of GVF profiles (Q=9.314x10^{-3} m^3/s and lined concrete)

Fig. 10 Optimal reproduction of GVF profiles (Q=9.314x10^{-3} m^3/s and lined concrete)

Estimated parameters

The bed roughness (n_1) varies from 0.027 to 0.034 as bed material/condition changes from lined concrete to gravel (d_{50}=20mm). The corresponding reported/Strickler's estimates are given in
Table 5.2 by using equation 1.2. It may be seen that optimal roughness estimates are higher than Strickler’s estimates.

Table 7 Reported/Strickler’s estimated optimal estimates for bed materials

<table>
<thead>
<tr>
<th>Bed material/condition</th>
<th>Reported/Strickler’s Estimation</th>
<th>Optimal estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>d_{50}=20mm</td>
<td>0.0247</td>
<td>0.034</td>
</tr>
<tr>
<td>d_{50}=6mm</td>
<td>0.0202</td>
<td>0.030</td>
</tr>
<tr>
<td>Lined concrete</td>
<td>0.013-0.015</td>
<td>0.027</td>
</tr>
</tbody>
</table>

The roughness coefficient of glass and GI sheet sides as optimized for various runs are presented in Table 8.

Table 8 Reported/Strickler’s estimates and optimal estimates for sides

<table>
<thead>
<tr>
<th>side</th>
<th>d_{50}</th>
<th>d_{50}=20mm</th>
<th>d_{50}=6mm</th>
<th>Lined concrete</th>
<th>Tabulated values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass</td>
<td>0.016</td>
<td>0.016</td>
<td>0.015</td>
<td>0.010</td>
<td></td>
</tr>
<tr>
<td>GI sheet</td>
<td>0.018</td>
<td>0.018</td>
<td>0.017</td>
<td>0.012</td>
<td></td>
</tr>
</tbody>
</table>

The estimated roughness coefficients satisfy the known inequality (n_2 < n_3) and are higher than the tabulated values. This establishes the credibility of the proposed model.

The optimal value of α (fitting parameter) ranges from 1.42 to 1.48, which differs from the reported value i.e. 1.5. The optimal value of α increases as the bed materials get finer.

Reproduction of observed profile

Computed GVF profiles corresponding to the optimal parameter values match quite well with corresponding observed profiles.

Variability of composite roughness
It can be observed that composite roughness reduces with increase in flow depth. Apparently because of increase in weightage of side resistance, the value of composite roughness increase.

**CONCLUSION**

This study was carried out to identify open channel flow parameters. Manning’s roughness coefficient and other parameters are estimated for different bed materials used (d50 =20mm grain size, 6mm grain size particles and Lined concrete bed materials). Also, based on the estimated value of Manning roughness coefficient and flow depths, GVF flow profile is identified.

An optimization method is applied to identify the parameters based on Manning formula for estimation of manning roughness coefficient and corresponding manning roughness parameters. This estimation invokes the data of observed GVF profiles and such accounts for different bed materials with the flow depth.

Experimental works is done to several sets of data monitored in Hydraulics Laboratory of Civil Engineering Department. The application led to the following conclusions;

i) The GVF profile computed on the basis of estimated parameters match quite closely with the corresponding observed profiles.

ii) Strickler’s formula under estimate the roughness due to the bed material.

iii) The following commonly used formula is calibrated for Manning coefficient estimation

\[ nc = \left( \frac{\sum_{i=1}^{N} n_i^\alpha P_i}{\sum_{i=1}^{N} P_i} \right)^{1/\alpha} \]

iv) The currently documented value of \( \alpha \) is 1.5. However, the present work reveals that it varies from 1.42 to 1.48. The value of \( \alpha \) generally decreases as the bed material gets coarser.

**REFERENCES**


