

A Novel Method for Power-Flow Solution of Radial Distribution Networks

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Abstract - With the perspective of the developing Smart grid notions, the upcoming distribution network will require repetitive and fast load flow solution that must be resolved as proficiently as possible in some applications frequently in distribution planning, automation, optimization of power system etc. This induces the continued exploration for precise and fast power flow procedures for distribution networks. In this paper a novel and effective method for power-flow solution of radial distribution networks is presented. This method is based on formulation of two matrices bus injection to branch current matrix and branch current to bus voltage matrix and load flow solution is obtained by simple multiplication of matrices in matlab platform. The proposed method is robust and proficient and effectiveness of proposed method is demonstrated by solving a 33-bus radial distribution system.

Keywords - Distribution load flow, distribution automation system, radial network, weakly meshed network

I. INTRODUCTION

Numerous platforms of real time applications in the region of distribution automation (DA), such as network optimization, Var planning, switching, state estimation, and so forth, require a robust and efficient load flow method [1]–[3]. Such a load flow method must be able to model the special features of distribution systems in sufficient detail. The well-known characteristics of an electric distribution system are

- Extremely large number of branches and nodes;
- Wide-ranging resistance and reactance values.

Those features cause the traditional load flow methods used in transmission systems, such as the Gauss-Seidel and Newton-Raphson techniques, to fail to meet the requirements in both performance and robustness aspects in the distribution system applications. In particular, the assumptions necessary for the simplifications used in the standard fast decoupled Newton-Raphson method [1] often are not valid in distribution systems. Therefore, a novel load flow algorithm for distribution systems is desired. To qualify for a good distribution load flow algorithm, all of the characteristics mentioned before need to be considered.

Several load flow algorithms specially designed for distribution systems have been proposed in the literature [2]–[8].

Some of these methods were developed based on the general meshed topology like transmission systems [2]–[5]. From those methods, the Gauss implicit matrix method [3] is one of the most commonly used methods; however, this method does not explicitly exploit the radial and weakly meshed network structure of distribution systems and, therefore, requires the solution of a set of equations whose size is proportional to the number of buses. Recent research proposed some new ideas on how to deal with the special topological characteristics of distribution systems [5]–[8], but these ideas require new data format or some data manipulations. In [5], the authors proposed a compensation-based technique to solve distribution load flow problems. Branch power flows rather than branch currents were later used in the improved version and presented in [6]. Since the forward/backward sweep technique was adopted in the solution scheme of the compensation-based algorithm, new data format and search procedures are necessary. Extension of the method, which emphasized on modeling unbalanced loads and dispersed generators, was proposed in [7]. In [8], the feeder-lateral based model was adopted, which required the “layer-lateral” based data format. One of the main disadvantages of the compensation-based methods is that new databases have to be built and maintained. In addition, no direct mathematical relationship between the system status and control variables can be found, which makes the applications of the compensation-based algorithm difficult.

The algorithm proposed in this paper is a “novel but classic” technique. The only input data of this algorithm is the conventional bus-branch oriented data used by most utilities. The goal of this paper is to develop a formulation, which takes advantages of the topological characteristics of distribution systems and solve the distribution load flow directly. It means that the time consuming LU decompositions and forward backward substitution of the Jacobian matrix or the admittance matrix, required in the traditional Newton Raphson and Gauss implicit Z matrix algorithms, are not necessary in the new development. Two developed matrices, the bus-injection to branch current matrix and the branch-current to bus-voltage matrix, and a simple matrix multiplication are utilized to obtain load flow solutions. The treatments for weakly meshed distribution systems are also included in this paper. The proposed method is very robust and very efficient compared to the conventional methods. Test results demonstrate the feasibility and validity of the proposed method.

II. UNBALANCED THREE-PHASE LINE MODEL

Fig. 1 shows a three-phase line section between bus i and j . The line parameters can be obtained by the method developed by

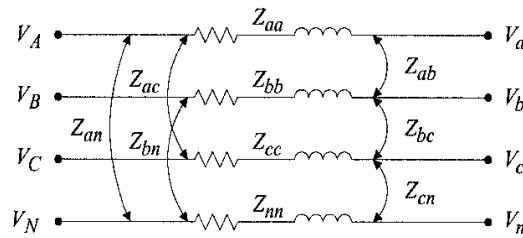


Fig 1. A three phase line section model.

Carson and Lewis [2]. A 4 X 4 matrix, which takes into account the self and mutual coupling effects of the unbalanced three phase line section, can be expressed as

$$[Z_{abcn}] = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} & Z_{an} \\ Z_{ba} & Z_{bb} & Z_{bc} & Z_{bn} \\ Z_{ca} & Z_{cb} & Z_{cc} & Z_{cn} \\ Z_{na} & Z_{nb} & Z_{nc} & Z_{nn} \end{bmatrix} \quad (1)$$

After Kron's reduction is applied, the effects of the neutral or ground wire are still included in this model as shown in (2)

$$[Z_{abc}] = \begin{bmatrix} Z_{aa-n} & Z_{ab-n} & Z_{ac-n} \\ Z_{ba-n} & Z_{bb-n} & Z_{bc-n} \\ Z_{ca-n} & Z_{cb-n} & Z_{cc-n} \end{bmatrix} \quad (2)$$

The relationship between bus voltages and branch currents in Fig. 1 can be expressed as

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} - \begin{bmatrix} Z_{aa-n} & Z_{ab-n} & Z_{ac-n} \\ Z_{ba-n} & Z_{bb-n} & Z_{bc-n} \\ Z_{ca-n} & Z_{cb-n} & Z_{cc-n} \end{bmatrix} \begin{bmatrix} I_{Aa} \\ I_{Bb} \\ I_{Cc} \end{bmatrix} \quad (3)$$

For any phases failed to present, the corresponding row and column in this matrix will contain null-entries.

III. ALGORITHM DEVELOPMENT

The proposed method is developed based on two derived matrices, the bus injection to branch current matrix and the branch-current to bus-voltage matrix, and equivalent current injections.

In this section, the development procedure will be described in detail.

For distribution networks, the equivalent-current-injection-based model is more practical [2]–[8]. For bus i the complex load S_i is expressed by

$$S_i = (P_i + jQ_i) \quad i = 1 \dots N. \quad (4)$$

And the corresponding equivalent current injection at the k^{th} iteration of solution is

$$I_i^k = I_i^r(V_i^k) + jI_i^i(V_i^k) = \left(\frac{P_i + jQ_i}{V_i^k} \right)^* \quad (5)$$

where V_i^k and I_i^k are the bus voltage and equivalent current injection of bus i at the k^{th} iteration, respectively. I_i^r and I_i^i are the real and imaginary parts of the equivalent current injection of bus i at the k^{th} iteration, respectively.

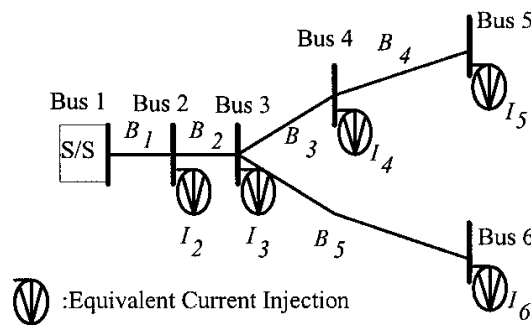


Fig 2. Simple distribution system

A. Relationship Matrix Developments

A simple distribution system shown in Fig. 2 is used as an example. The power injections can be converted to the equivalent current injections by (5), and the relationship between the bus current injections and branch currents can be obtained by applying Kirchhoff's Current Law (KCL) to the distribution network. The branch currents can then be formulated as functions of

equivalent current injections. For example, the branch currents B_1 , B_2 and B_5 , can be expressed by equivalent current injections as in equation (6)

$$\begin{aligned} B_1 &= I_2 + I_3 + I_4 + I_5 + I_6 \\ B_2 &= I_4 + I_5 \\ B_5 &= I_6 \end{aligned} \tag{6}$$

Therefore the relationship between the bus current injections and branch currents can be expressed as in (7a)

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} \tag{7a}$$

Equation (7a) can be expressed in general form as

$$[B] = [BIBC][I] \tag{7b}$$

Where BIBC is the bus injection to branch current (BIBC) matrix.

The constant BIBC matrix is an upper triangular matrix and contains values of 0 and +1 only.

The relationship between branch currents and bus voltages as shown in Fig. 2 can be obtained by (3). For example, the voltages of bus 2, 3, and 4 are

$$V_2 = V_1 - B_1 Z_{12} \tag{8a}$$

$$V_3 = V_2 - B_2 Z_{23} \tag{8b}$$

$$V_4 = V_3 - B_3 Z_{34} \tag{8c}$$

Where V_i is the voltage of bus, and Z_{ij} is the line impedance between bus i and bus j .

Substituting (8a) and (8b) into (8c), (8c) can be rewritten as

$$V_4 = V_1 - B_1 Z_{12} - B_2 Z_{23} - B_3 Z_{34} \tag{9}$$

From (9), it can be seen that the bus voltage can be expressed as a function of branch currents, line parameters, and the sub-station voltage. Similar procedures can be performed on other buses; therefore, the relationship between branch currents and bus voltages can be expressed as

$$\begin{bmatrix} V_1 \\ V_1 \\ V_1 \\ V_1 \\ V_1 \end{bmatrix} - \begin{bmatrix} V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} Z_{12} & 0 & 0 & 0 & 0 \\ Z_{12} & Z_{23} & 0 & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & Z_{45} & 0 \\ Z_{12} & Z_{23} & 0 & 0 & Z_{36} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} \tag{10a}$$

Equation 10(a) can be written as:-

$$[\Delta V] = [BCBV][B] \tag{10b}$$

where BCBV is the branch-current to bus-voltage (BCBV) matrix.

B. Building Formulation Development

Observing (7), a building algorithm for BIBC matrix can be developed as follows:

Step 1) For a distribution system with m branch section and n bus, the dimensions of the BIBC matrix is $m \times (n - 1)$.

Step 2) If a line section is located between bus i and bus j , copy the column of the i -th bus of the BIBC matrix to the column of the j -th bus and fill a +1 to the position of the i -th row and the j -th bus column.

Step 3) Repeat procedure (2) until all line sections are included in the BIBC matrix. From (10), a building algorithm for BCBV matrix can be developed as follows.

Step 4) For a distribution system with m branch section and n bus the dimensions of the BCBV matrix is $(n-1) \times m$.

Step 5) If a line section is located between bus i and bus j , copy the row of the i -th bus of the BCBV matrix to the row of the j -th bus and fill the line impedance to the position of the j -th bus row and the i -th column.

Step 6) Repeat procedure (5) until all line sections are included in the BCBV matrix.

The algorithm can easily be expanded to a multiphase line section or bus. For example, if the line section between bus i and bus j is a three phase line section, the corresponding branch current B_i will be a 3 X 1 vector and the +1 in the BIBC matrix will be a 3X3 identity matrix. Similarly, if the line section between bus i and bus j is a three-phase line section, the Z_{ij} in the BCBV matrix is a 3X3 impedance matrix as shown in (2).

It can also be seen that the building algorithms of the BIBC and BCBV matrices are similar. In fact, these two matrices were built in the same subroutine of our test program. Therefore, the computation resources needed can be saved. In addition, the building algorithms are developed based on the traditional bus-branch oriented database; thus, the data preparation time can be reduced and the proposed method can be easily integrated into the existent DA.

C. Solution Technique Developments

The BIBC and BCBV matrices are developed based on the topological structure of distribution systems. The BIBC matrix represents the relationship between bus current injections and branch currents. The corresponding variations at branch currents, generated by the variations at bus current injections, can be calculated directly by the BIBC matrix. The BCBV matrix represents the relationship between branch currents and bus voltages. The corresponding variations at bus voltages, generated by the variations at branch currents, can be calculated directly by the BCBV matrix. Combining (7b) and (10b), the relationship between bus current injections and bus voltages can be expressed as

$$[\Delta V] = [BCBV][BIBC][I] = [DLF][I]. \tag{11}$$

And the solution for distribution load flow can be obtained by solving (12) iteratively

$$I_i^k = I_i^0(V_i^k) + jI_i^1(V_i^k) = \left(\frac{P_i + jQ_i}{V_i^k} \right)^* \tag{12a}$$

$$[\Delta V^{k+1}] = [DLF][I^k] \tag{12b}$$

$$[V^{k+1}] = [V^0] + [\Delta V^{k+1}]. \tag{12c}$$

According to the research, the arithmetic operation number of LU factorization is approximately proportional to N^3 . For a large value of N, the LU factorization will occupy a large portion of the computational time. Therefore, if the LU factorization can be avoided, the load flow method can save tremendous Computational. From the solution techniques described before, the LU decomposition and forward/backward substitution of the Jacobian matrix or the Y admittance matrix are no longer necessary for the proposed method. Only the **DLF matrix** is necessary in solving load flow problem. Therefore, the proposed method can save considerable computation resources and this feature makes the proposed method suitable for online operation.

IV. TREATMENTS FOR WEAKLY MESHED NETWORKS

Some distribution feeders serving high-density load areas contain loops created by closing normally open tie-switches. The proposed method introduced before can be extended for “weakly-meshed” distribution feeders.

A. Modification for BIBC Matrix

Existence of loops in the system does not affect the bus current injections, but new branches will need to be added to the system. Fig. 3 shows a simple case with one loop. Taking the new branch current into account, the current injections of bus 5 and bus 6 will be

$$I'_5 = I_5 + B_6$$

$$I'_6 = I_6 + B_6$$

(13)

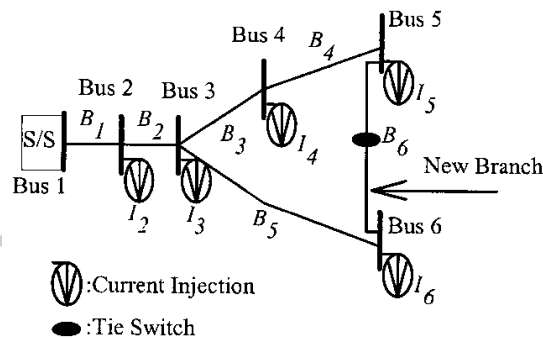


Fig 3. Simple distribution system with one loop.

The BIBC matrix will be

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 + B_6 \\ B_6 \end{bmatrix}. \tag{14a}$$

Equation (14a) can be rewritten as

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} B_6 \\ -B_6 \end{bmatrix}. \tag{14b}$$

And the modified BIBC matrix can be obtained

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ B_6 \end{bmatrix} \quad (15a)$$

The general form for the modified BIBC matrix is

$$\begin{bmatrix} B \\ B_{new} \end{bmatrix} = [BIBC] \begin{bmatrix} I \\ B_{new} \end{bmatrix} \quad (15b)$$

The building algorithm of Step 2) for BIBC matrix can be modified as follows:

Step 2a)—If a new branch (Bk) makes the system become meshed (the new branch is between bus i and j), copy the elements of the i^{th} bus column to the k^{th} column and minus the elements of the j^{th} bus column. Finally, fill a +1 value to the position of the k^{th} row and the k^{th} column.

B. Modification for BCBV Matrix

Considering the loop shown in Fig. 3, KVL for this loop can be written as

$$Z_{34}B_3 + Z_{45}B_4 + Z_{56}B_6 - Z_{36}B_5 = 0. \quad (16)$$

Combining (16) and (10a), the new BCBV matrix is

$$\begin{bmatrix} V_1 \\ V_1 \\ V_1 \\ V_1 \\ V_1 \\ 0 \end{bmatrix} - \begin{bmatrix} V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{12} & 0 & 0 & 0 & 0 & 0 \\ Z_{12} & Z_{23} & 0 & 0 & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & 0 & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & Z_{45} & 0 & 0 \\ Z_{12} & Z_{23} & 0 & 0 & Z_{36} & 0 \\ 0 & 0 & Z_{34} & Z_{45} & Z_{36} & Z_{56} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \end{bmatrix} \quad (17a)$$

The general form for the modified BCBV matrix is

$$\begin{bmatrix} \Delta V \\ 0 \end{bmatrix} = [BCBV] \begin{bmatrix} B \\ B_{new} \end{bmatrix} \quad (17b)$$

The building algorithm of Step 5) for the BCBV matrix can be modified as follows:

Step 5a)—If a new branch (Bk) makes the system become meshed, adds a new row to the original BCBV matrix by KVL.

C. Modification for Solution Techniques

Substituting (15) and (17) into (11), (11) can be rewritten as

$$\begin{bmatrix} \Delta V \\ 0 \end{bmatrix} = [BCBV][BIBC] \begin{bmatrix} I \\ B_{new} \end{bmatrix} = \begin{bmatrix} A & M^T \\ M & N \end{bmatrix} \begin{bmatrix} I \\ B_{new} \end{bmatrix} \quad (19)$$

Applying Kron’s Reduction to (19), the modified algorithm for weakly meshed networks can be expressed as

$$\begin{aligned} \Delta V &= [A \quad M^T N^{-1} M][I] \\ &= [DLF][I]. \end{aligned} \quad (20)$$

Note that except for some modifications needed to be done for the BIBC, BCBV, and DLF matrices, the proposed solution techniques require no modification; therefore, the proposed method can obtain the load flow solution for weakly meshed distribution systems efficiently.

V. CONCLUSION

In this paper, a direct approach for distribution load flow solution was proposed. Two matrices, which are developed from the topological characteristics of distribution systems, are used to solve load flow problem. The BIBC matrix represents the relationship between bus current injections and branch currents, and the BCBV matrix represents the relationship between branch currents and bus voltages. These two matrices are combined to form a direct approach for solving load flow problems. The time-consuming procedures, such as the LU factorization and forward/backward substitution of the Jacobian matrix or Y admittance matrix, are not necessary in the proposed method. The ill-conditioned problem that usually occurs during the LU factorization of the Jacobian matrix or Y admittance matrix will not occur in the proposed solution techniques. Therefore, the proposed method is both robust and efficient. Test results show that the proposed method is suitable for large-scale distribution

systems. Other issues involved in the distribution system operation, such as multiphase operation with unbalanced and distributed loads, voltage regulators, and capacitors with automatic tap controls, will be discussed in future work.

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