Time Domain Electric Arc Furnace Modeling - A Survey

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Abstract—Power quality is becoming a more concern of today’s power system engineer due to rapid growth of non-linear loads in distribution network. Electrical Arc Furnace (EAF) is one of the responsible causes for deteriorating power quality in the distribution network. Hence electric arc furnace model is needed to study and to analyze the power quality in the distribution network. This paper presents a survey of various EAF models of electric arc furnace to study power quality problems. Various time domain models of EAF are surveyed and studied to describe the behavior of the EAF for all of the operating conditions.

Index Terms—Electric arc furnace, harmonics, harmonic distortion, arc

I. NOMENCLATURE

i = Arc current
v = Arc voltage
σ = Arc conductance
E₀ = Momentarily constant steady state arc voltage
θ = Arc time constant
θ₀ = Constant
θ₁ = Constant
α = Constant
P₀ = Momentarily power loss
I₀ = Transition current
σ_min = Minimum conductance
ITHD = Total Current Harmonic distortion
VTHD = Total Voltage Harmonic distortion
Q = Reactive power to be generated by the filter at fundamental frequency

II. INTRODUCTION

The increasing popularity of EAF in metallurgical industries to melt scrap causes significant impacts on power system and on electrical power quality. EAF is one of the responsible source for deteriorating the power quality in the connected network [1-3]. The EAF is inherently non-linear, time-variant load and it can cause power quality problems such as current-voltage harmonics, voltage flicker and voltage unbalance. Odd and even harmonic currents are generated by EAF operations. These harmonic currents, when circulated in the electric network can generate harmonic voltages which in turn can affect other users connected in the distribution network.

The important issue in the modeling of the EAF is the simulation of arc. There are several methods used to describe the electric arc [1-4, 7-8]. On the basis of actual measured samples of an electric arc in several functioning cycles of EAF, different operating points are generated in the form of statistical probability, corresponding to hidden Markov theory in [1]. This requires actual measurement of an electric arc. The time domain methods based on the differential equations are also presented [2]. Variation of power transmitted to the load by the arc furnace during the cycle of operation is considered in [3]. Comparison of EAF modeling in time domain and frequency domain shows that time domain is more useful in studying the EAF [4, 8].

III. EAF MODELING AS NON-LINEAR LOAD

The arc melting process is a non-stationary stochastic process, so it is difficult to make an appropriate model for an arc furnace load. The dynamic specifications of the EAF at any instant of time are affected by conditions of the furnace at that time and previous instances of the time. The reason for that is when arc is created, the sudden change in the electrons, ions and gas temperature (that may occur due to sudden change of current) is impossible. That means, the sudden change of the current will not lead to sudden change of the arc characteristic. Thus the phenomenon happens gradually. In brief, the time response of an EAF depends on [6, 11-12],

1. Melting or refining materials
2. Melting stage
3. Positions of electrodes (arc length)
4. Electrode arm control
5. Topology of external circuit

Modeling is based on the estimation of the voltage and current of the electric arc

As shown in Figure 1, the electric arc consist four major Areas:

1. Area 1,  \( \frac{di_{arc}}{dt} > 0 \),  \( v_{arc}, i_{arc} > 0 \)
2. Area 2,  \( \frac{di_{arc}}{dt} < 0 \),  \( v_{arc}, i_{arc} > 0 \)
3. Area 3,  \( \frac{di_{arc}}{dt} < 0 \),  \( v_{arc}, i_{arc} < 0 \)

Fig. 1 The actual V-I Model of EAF
4. Area 4, \(\frac{di_{arc}}{dt} > 0, v_{arc} \cdot i_{arc} < 0\)

IV. VARIOUS EAF MODELS

A. Model 1

This work presents a new time domain Controlled Voltage Source Model (CVSM) for an AF using MATLAB [5, 9, 13]. The developed model is based on a linear approximation of the VI characteristic of the arc furnace load. Active power consumed by the load is also considered in the proposed load model of AF, thus making the proposed load model dependent on the operating conditions of the load.

\[
i_1 = \frac{V_{ig}}{R_1}
\]

(1)

\[
i_2 = V_{ex} - V_{ig} \left(1 - \frac{R_2}{R_1}\right)
\]

(2)

\[
v = \begin{cases} iR_1; & 0 \leq i \leq i_1 \\ iR_2 + V_{ig} \left(1 - \frac{R_2}{R_1}\right); & i_1 < i \leq i_2 \end{cases}
\]

(3)

Fig. 2 VIC of model 1

Where \(v\) is the voltage, \(i\) is the current, \(R_1\) and \(R_2\) are the slopes of segments OA and AB respectively and \(V_{ig}\) and \(V_{ex}\) are the ignition and extinction voltages of the arc respectively. \(i_1\) and \(i_2\) are the currents corresponding to the ignition and extinction voltages respectively.

B. Model 2

The EAF is considered as a controlled voltage source based on the linear approximation of V-I characteristics during arc ignition and arc extinction region [8]. The arc starts igniting when the arc current reaches a certain value called ignition when the arc current reaches a certain value called ignition current \(i_1\) and arc voltage rises to arc ignition voltage \(V_{ig}\). This process continues until the arc voltage reaches its extinction voltage \(V_{ex}\). The arc extinguishes when arc voltage drops below arc extinction voltage \(V_{ex}\)

\[
i_1 = \frac{V_{ig}}{R_1}
\]

(4)

\[
i_2 = \frac{V_{ex} - V_{ig}}{R_2} \left(\frac{1}{R_2} - \frac{1}{R_1}\right)
\]

(5)

Here, \(i_1\) represents the current where arc starts igniting from extinction, and \(i_2\) represents the current where the arc starts extinguishing. The relationship between the arc voltage and arc current is given by:

\[
v = \begin{cases} iR_1; & -i_1 \leq i \leq i_1 \\ iR_2 + V_{ig} \left(1 - \frac{R_2}{R_1}\right); & i_1 < i \leq i_2 \\ iR_2 - V_{ig} \left(1 - \frac{R_2}{R_1}\right); & -i_2 \leq i < -i_1 \end{cases}
\]

(6)

Where \(R_1\) and \(R_2\) are the slope of segment OA and AB shown in Fig. 4.

C. Model 3

In this model, the arc melting process is divided into three sections:

1. The arc is from extinction to re-ignition: The voltage magnitude increases from extinction voltage \(V_{ex}\) to re-ignition voltage \(V_{ig}\), the arc furnace acts as resistance, and the arc current changes its polarity from \(-i_3\) to \(i_1\).

2. Beginning of arc melting process: There is a sudden voltage drop across the electrode, thus the arc voltage decreases from \(V_{ig}\) to \(V_{st}\), and the arc current has a little increase from \(i_1\) to \(i_2\). The voltage drop is approximated in an exponential way.

3. Normal arc melting process: The arc voltage drops slowly and smoothly from \(V_{st}\) to \(V_{ex}\). Since the melting process spans most of the half cycle, the mean value is assumed to be \(V_{m}\). Because the arc current increases to its maximum before it drops \(i_3\), the VIC is divided into current increasing and decreasing parts, and expressed as [5,9]:

\[ v = \begin{cases} 
{iR_1}; & i < i_1 & \text{and} \frac{di}{dt} > 0 \text{ or } i < i_2 & \text{and} \frac{di}{dt} < 0 \\
{V_{st} + (V_{ig} - V_{st}) \exp \left( \frac{(i - i_1)}{i_4} \right)}; & i_1 < i < i_2 \\
{V_{st} + (i - i_2)R_2}; & i > i_2 \text{ and } \frac{di}{dt} > 0 \\
{V_{ex} + (i - i_3)R_3}; & i < i_3 \text{ and } \frac{di}{dt} < 0 
\end{cases} \]  

(7)

Where \( R_1 \), \( R_2 \), \( R_3 \) and \( R_4 \) are the corresponding slopes of each section, and

\[ i_1 = \frac{V_{ig}}{R_1} \]
\[ i_2 = 3i_1 \]
\[ i_3 = \frac{V_{ex}}{R_1} \]
\[ i_4 = 1.5i_1 \]  

(8)

In this model, the mean voltage level \( V_m \) is a function of the arc length, is used to reflect the operating condition. \( V_{ig} \), \( V_{st} \) and \( V_{ex} \) are also considered to be proportional to \( V_m \).

D. Model 4

Model 4 is developed based on linear approximation of V-I characteristics of the arc furnace, as shown in Fig. The expression of arc voltage is given by [5,9]:

\[ v = \begin{cases} 
{iR_1}; & 0 \leq i \leq i_1 & \text{and} \frac{di}{dt} > 0 \text{ and } 0 \leq i \leq i_4 & \text{and} \frac{di}{dt} < 0 \\
{R_2(i - i_1) + V_1}; & i_1 < i \leq i_2 & \text{and} \frac{di}{dt} > 0 \\
{R_3(i - i_2) + V_2}; & i_2 < i \leq i_3 & \text{and} \frac{di}{dt} < 0 \\
{R_4(i - i_3) + V_3}; & i_3 < i \leq i_4 & \text{and} \frac{di}{dt} < 0 
\end{cases} \]  

(9)

\( R_1 \) represents the resistance during pre-ignition period, and \( R_2 \) represents the resistance when arc is established and the arc current is increasing. \( R_3 \) represents the arc resistance during ignition period when the arc current is decreasing, and \( R_4 \) represents the arc resistance before the arc extinguishes.

The expressions for \( i_1 \), \( i_2 \), \( i_3 \) and \( i_4 \) are as follows:

\[ i_1 = \frac{V_1}{R_1} \]
\[ i_2 = f(V_1, R_2) \]
\[ i_3 = f(V_2, R_3) \]
\[ i_4 = \frac{V_3}{R_4} \]  

(10)

E. Model 5

In this model the VIC of the EAF is considered in the form \( V = V(I) \) as [5,14]:

\[ V(I) = \left[ V_T + \frac{C_{i,d}}{D_{i,d} + |I|} \right] \text{signum}(I) \]  

(11)

Where \( I \) and \( V \) are the arc current and voltage of a given phase, respectively. \( V_T \) is the magnitude of the voltage threshold to which the voltage approaches as the current increases. This voltage is dependent on the arc length. Constants \( C_{i,d} \) and \( D_{i,d} \) are corresponding to the arc power and arc current respectively. These constant take different values which depend on the sign of derivative of the arc current. There are two paths to increase or decrease of current. The first path is related to the increasing state of the current and the second path is associated with decreasing state of the current. In this regard, the constants \( C \) and \( D \) are classified into two groups. The constants for the first group (first path) are \( C_a \) and \( D_a \). Also constants of the second group (second path) are \( C_b \) and \( D_b \). Since (1) is similar to the hyperbolic function, it is named a hyperbolic model.

F. Model 6

This is Mayr’s arc model. The expression of arc voltage and current are given by [13-14]:

\[ i_a = \sqrt{\frac{2}{\tau_a}} \sin \alpha \]

The arc voltage is expressed as:

\[ V_a = \frac{2V_0 \sin \alpha}{1 - \frac{\sin(2\alpha + \psi_a)}{\sqrt{1 + (2\alpha \tau_a)^2}}} \]  

(12)

Where, \( V_0 = \frac{p_{rc}}{\sqrt{2I}} \), \( \tan \psi_a = \frac{1}{2\alpha \tau_a} \)

\( \tau_a \) is the arc time constant and is the most important parameter that governs V-I characteristic. \( p_{rc} \) is the arc column power at the moment of interruption. V-I characteristics of Model 6 is shown in Fig. 8.

G. Model 7

The VIC of the arc in this model is approximated by an exponential function as follows [7]:

\[ V(I) = V_T \left( 1 - e^{-|I|/I_0} \right) \text{signum}(I) \]  

(13)

In the equation describing this model, a constant current \( I_0 \) is employed to model the steepness of positive and negative currents, and an exponential function is used to describe the VIC of the arc.

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<th>TABLE I. EAF MODEL PARAMETERS</th>
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V. SIMULATION RESULTS

To compare the different arc furnace models, a simple arc furnace system is studied in single phase. The system configuration is shown in Fig 3.

![Test circuit of electric arc furnace connected supply system](image)

In figure 3, the system impedance is represented as, bus PCC represents the point of common coupling, and bus AF is the low voltage side of the transformer whose impedance is given as. Simulation diagram is shown in Fig 8 and the system parameters and the parameters of each model are presented in Table I.

![Simulation results](image)
VI. DISCUSSION

In whole models mathematical relationship for perfect performance cycle is obtained with assumption that V-I curve is symmetric with respect to origin of axis. So all the models only generates odd harmonics. However actual arc furnace generates even harmonics.

Referring to Fig1, the actual V-I characteristic of arc furnace has three parts. In the first period, the arc begins to reignite from extinction. When the arc voltage reaches to the re-ignition voltage, the equivalent circuit acts as an open circuit. In the second period, the arc is established and the arc the voltage drops from to increase the electrical conductivity of the arc (AB path). During the third part, the arc begins to extinguish. The arc voltage continues to drop smoothly (BO path). Figures 4 to 10 show the V-I along with the arc current and arc voltage waveform at PCC.

Comparison of simulation results of various arc furnace models show that the models 1, 2 and 7 are not able to simulate the third part so the response of models 1 to 6 is different from the actual performance of arc furnace. From the results of simulation, we conclude that the models 3 and 6 are more accurate present better results.
VII. CONCLUSIONS

Arc furnace is usually modeled in frequency and time domain. Modeling in frequency domain needs some measurements that in most cases is not provided that is why time domain modeling is more favorable. In this study, different models for arc furnace in the time domain are reviewed. The modeling is based on V-I characteristic approximation.

In this paper, seven such major models of arc furnace are studied from power quality point of view. In these models arc furnace is modeled as a current controlled voltage source. The models can be applied to predict response of the arc furnace, during the melting cycle. The models are simulated using MATLAB on a proto-type system.

In whole models mathematical relationship for perfect performance cycle is obtained with assumption that V-I curve is symmetric with respect to origin of axis. So all the models only generates odd harmonics. However actual arc furnace generates even harmonics.

Each model is suitable to analyze the harmonics content of the arc current and the arc voltage, and for the voltage distortion analysis. However, the voltage distortion can be varied as the arc resistance varies randomly. Each model can be incorporated in metallurgical plants and dynamic response can be achieved.

VIII. REFERENCES


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