PMU Placement for Power System Observability using Integer Linear Programming

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***Abstract*—an integer linear programming based methodology for optimal placement of PMU in a given power network for full observability of that network is presented in this paper. First conventional complete observability of the given network is formulated and then, zero injection bus constraints are added in previous formulation. The results from conventional and modified formulation are than compared. Moreover minimum PMU placement problem may have multiple solutions, so to decide best one, two indices are proposed, BOI and SORI, where BOI is Bus Observability Index and SORI is System Observability Redundancy Index. Results on 9 bus, IEEE 14 bus systems are presented.**

***Index Terms*—Integer Linear Programming(ILP), Phasor Measurement Unit (PMU), Global positioning satellite(GPS), Zero injection measurements.**

I. Introduction

Phasor Measurement Units provide time synchronized phasor measurements in power system [1]. PMUs were first introduced in 1980's and since then have become a mature technology with many applications [3]. Synchronization of voltage and current is achieved by time stamping of voltage and current waveforms using common synchronizing signal available from GPS system [1]. The common time reference used for time stamping in voltage and current waveforms at each bus in a given system is GPS signal which has accuracy of 1µs[2]. This is how ac quantities are measured, time stamped using a common reference and converted to phasors (a sinusoidal quantity converted in phasor is represented by its magnitude and phase angle). Using data collected from PMU can improve performance of monitoring of given power network.

The first objective of this paper is to find out optimal location of PMU in power network. As most power networks are interconnected and having large number of buses, continuous measurement of all parameters of a transmission network for monitoring is required. But PMU cannot be placed on every bus of a large power network as cost of PMU is very high. So it is neither economical nor necessary to place a PMU on each and every bus.

There are mainly two types of measurements derived from PMU. One is direct measurement and other is pseudo measurement. The bus where PMU is placed, voltage and current phasor of that bus can be obtained plus voltage and current phasors of all incident branches can be determined. That is direct measurement. If voltage and current phasor of one end of a branch is known than voltage phasor of other end of the branch can be obtained. If voltage phasor of both end of a branch is known than current phasor of that branch can be obtained. That is pseudo measurement.

So using these two measurements we do not have to place PMU on each bus. Thus finding optimal places from which whole system is observed is very much important. A 9 bus system is taken as a test system through this paper and after obtaining results in these system results of IEEE 14 bus are derived.

II. Problem Formulation

The objective of PMU placement problem is to find minimum number of locations from where whole system becomes topologically observable. The problem is formulated as follows:

$$\text{Minimize} \sum_{j \in I} u_j \qquad (2.1)$$

Subject to

$$f_i \geq 1 \ , \forall_i \in I \qquad (2.2)$$

Where

$$f_i = \sum_{j \in I} a_{ij} u_j, \qquad \forall_i \in I$$

(2.3)

Using this formulation optimal PMU placement solution is obtained for a 9 bus system shown here.

Where

i and j = indices of bus

I = set of buses

u_j = binary decision variable that is equal to 1 if PMU is installed at bus j and 0 otherwise

f_i = observability function of bus i

a_{ij} = binary connectivity parameter between bus i and j

Also,

a_{ij} = 1, if i=j
=1, if i and j are connected
=0, otherwise

The observability of a bus depends on the installation of PMU on that bus or on one of its incident bus [2].

Consider a nine bus system shown here and following the proposed formulation.

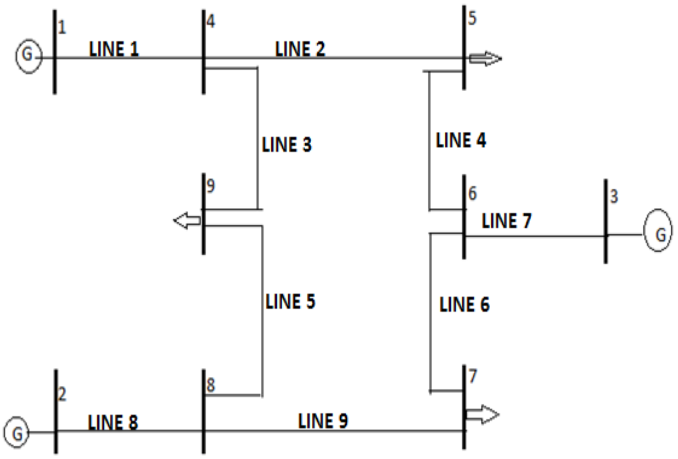


Fig 1. 9 bus test system

So according to the proposed formulation the objective function and constraints will be as follows.

Minimize u1+u2+u3+u4+u5+u6+u7+u8+u9

Subject to
f1 = u1+u4 >=1
f2 = u2+u8 >=1
f3 = u3+u6 >=1
f4 = u1+u4+u5+u9 >=1
f5 = u4+u5+u6 >=1
f6 = u3+u5+u6+u7 >=1
f7 = u6+u7+u8 >=1
f8 = u2+u7+u8+u9 >=1
f9 = u4+u8+u9 >=1

Using integer linear programming the optimal solution i.e. optimal location in given network where we can place a PMU and make all buses observable are u2, u4 and u6.

Now referring Fig.1, if we place a PMU on bus 2, bus 8 will also become observable as it is incident to bus 2. Similarly if a PMU is placed on bus 4 than, the three incident buses b1, b5 and b9 will also be observable and placing a PMU on bus 6 the incident buses to bus 6 which are b3, b5 and b7 become observable.

Hence in a nine bus system we do not have to put nine PMU devices to collect phasor data from those buses instead placing 3 PMU we can monitor whole system.

Same results are obtained for IEEE 14 bus system.

III. MODELLING OF ZERO INJECTIONS

Zero injection buses in a system are those buses which are not connected to any generator or directly to any load the just transfer power from one point to other. So zero injection corresponds to transhipment nodes. At zero injection bus no current is injected into the system. If zero injection buses are also modelled in the PMU placement problem, the total number of PMUs can further be reduced [1].

The zero injection bus rules for accessing network observability are:

(1) When buses, which are incident to an observable zero injection bus, are all observable accept one, the unobservable bus will also be identified as observable applying the KCL at zero injection bus.

(2) When buses incident to an unobservable zero-injection bus are all observable, the zero injection bus will also be identified as observable by applying KCL at zero injection bus.

These two conditions can be combined into one by indicating that among a zero-injection bus and its incident buses, a single bus can be made observable by making the others observable.[2] Using this simplification, the proposed formulation considering the effect of zero injection bus is presented as:

$$f_i \geq 1, \forall_i \in I \quad (3.1)$$

Where

$$f_i = \sum_{j\in I} a_{ij}\, u_j + \sum_{j\in I} a_{ij}\, u_j y_{ij} \quad \forall_i \in I \quad (3.2)$$

$$\sum_{i\in I} a_{ij}\, y_{ij} = z_j, \quad \forall_j \in I$$

(3.3)

Here,

y_{ij} = auxiliary binary variable of bus i and j.

z_j = zero injection parameter of bus j

The above equations are same as previous but y_{ij} which is auxiliary binary variable and z_j which is zero injection parameter of bus j is added.

Also,

Z_j = 1, if j is zero injection bus
= 0, otherwise.

Again above formulation is applied to same nine bus system. Here in the system three zero injection buses are taken. Bus 4, 6 and 8 are zero injection buses in our system. So the equations are as below:

Minimize u1+u2+u3+u4+u5+u6+u7+u8+u9

Subject to
f1 = u1+u4+y14 >= 1
f2 = u2+u8+y28 >= 1
f3 = u3+u6+y36 >= 1
f4 = u1+u4+u5+u9+y44 >=1
f5 = u4+u5+u6+y54+y56 >=1
f6 = u3+u5+u6+u7+y66 >= 1
f7 = u6+u7+u8+y76+y78 >= 1
f8 = u2+u7+u8+u9+y88 >= 1
f9 = u4+u8+u9+y98 >= 1

And the equality constraints are,
y11+y41 =0
y22+y82 =0
y33+y63 =0
y14+y44+y54+y94 =0
y45+y55+y65 =0
y36+y56+y66+y76 =0
y67+y77+y87 =0
y28+y78+y88+y98 =0
y49+y89+y99 =0

Using above objective function, equality and inequality constraints in integer linear programming the solution for nine bus system is u6 and u9. That means if two PMUs are placed on two buses bus 6 and bus 9 the whole nine bus network becomes observable.

It can be justified from the above diagram that if one PMU is placed on bus 6, the voltage and current phasor of bus 6 can be obtained. And as buses 5, 3 and 7 are incident to bus 6, by pseudo measurement the phasors of theses three buses can also be obtained i.e. these three buses are observable by one PMU on bus 6.

Similarly, as from our result using ILP solver one PMU is placed on bus 9, so phasor data of bus nine is obtained. Now as buses 4 and 8 are incident to bus 9 directly, phasor data of these buses are measured using pseudo measurement. Moreover both the buses bus 4 and 8 are zero injection bus, and are observable, so as bus incident to any observable zero injection bus becomes observable bus 1 and bus 2 will also be observable. Here bus 1 is incident to an observable zero injection bus 4 and bus 2 is incident to an observable zero injection bus 8.

If both results are compared, without using zero injection bus and after using zero injection bus, the system which was observable using three PMUs before, becomes observable using only two PMUs after modelling zero injection constraints in previous formulation. The system taken here as a test system is a nine bus system and a difference of 1 PMU is derived. But actual systems are interconnected and complex having number of buses, so applying constrains using zero injection bus gives us far better results. And PMU is a costly device so it is very economical if with less number of PMU our system becomes observable.

The results for IEEE 14 bus are obtained using zero injection bus constraints.

IV. MAXIMIZING REDNDANCY IN OBSERVABILITY

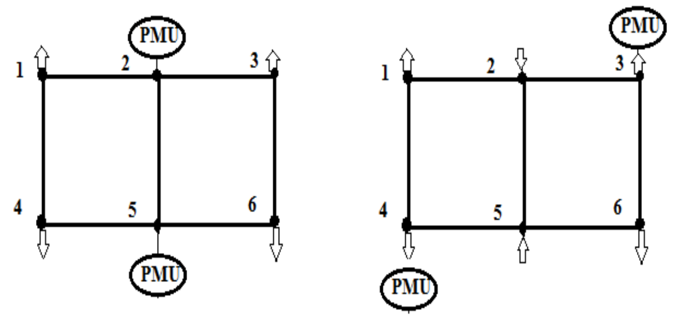
The optimal PMU placement problem has number of solutions, than the problem of superiority of particular solution arises. So to find out best solution amongst the others two indices are introduced. BOI is bus observability index. We can consider BOI as a performance indicator on quality of optimization. Consequently maximum bus observability is limited to maximum connectivity factor plus one.

$$\beta_i \leq \eta_i + 1 \qquad (4.1)$$

The other indices is SORI i.e. system observability redundancy index which is sum of bus observability for all buses.

$$\sum_{i=0}^{n} \beta_i = \gamma \qquad (4.2)$$

Consider a six-bus system shown in Fig. 2. It is seen that a minimum of two PMUs are required to ascertain system observability. Consider two such optimal solutions shown in Fig.2



2(a) 2(b)

Fig 2: Six Bus System

The system shown in fig 2(a) is observable by two PMUs placed at buses 2 and 5. BOI for bus 1 to 6 are 1, 2,1,1,2 and 1. i.e. bus 1 is observed by 1 PMU bus 2 is observed by two PMUs, bus 3 is observed by again 1 PMU and so on.

SORI for system 2(a) = 1+2+1+1+2+1 = 8.

The system shown in fig 2(b) is observable by two PMUs placed at buses 3 and 4. BOI for bus 1 to 6 are 1, 1,1,1,1 and 1. All the buses are observed by 1 PMU.

SORI = 1+1+1+1+1+1 = 6.

So, we can say that system 2(a) is more redundant than system 2(b). And we can select bus 2 and bus 5 as optimum places for PMU placement problem. We can follow the same exercise for 9 bus system IEEE 14 bus system.

Consider Fig 3 and Fig 4.

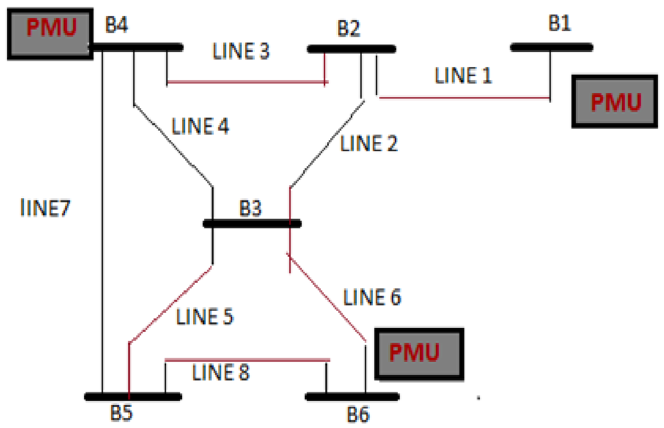


Fig 3: optimal placement of PMU in 6 bus system

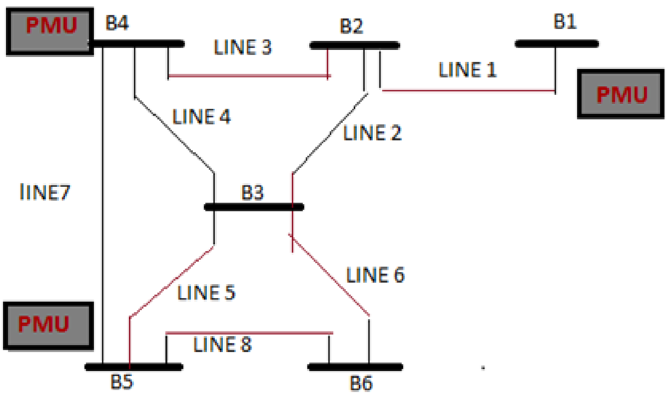


Fig 4: optimal placement of PMU in 6 bus system

As shown in fig.3 and fig.4 both the systems are observable by three PMUs. So it is to be decided which one is more redundant and can be selected for operation.

In fig 3 BOI for buses 1 to 6 is 1,2,2,1,2,1.

SORI = 1+2+2+1+2+1 = 9

In fig 4 BOI for buses 1 to 6 is 1,2,2,2,2,1

SORI = 1+2+2+2+2+1 = 10.

So the system shown in fig 4 is more redundant.

V. CASE STUDIES.

Test system	No. Of PMU	Placement buses
9 bus	3	2,4,6
14 bus	4	2,7,11,13

Table 1: Results of 9 and 14 bus system

Test system	No. Of PMU	Placement buses
9 bus	2	6,9
14 bus	3	2,6,9

Table 2: Results of 9 and 14 bus system including zero injection bus

VI. CONCLUSION

Basic idea of PMU and GPS and synchrophasor is presented. In a large power system placement of a PMU at every bus is not possible so optimum placement by which whole network remains observable is very necessary. Thus we get voltage and current phasor measurement of all the buses present in the network from the selected optimally placed PMUs.

To find this optimum places there are many methods. Out of these methods integer linear programming method is used in this report. As the objective function and constraints are linear the method is very simple to understand. All the optimization problems are solved using TORA solver which uses branch and bound algorithm for the optimization problem. Effect of zero injection bus is also presented so that more optimum result is derived.

VII. REFERENCES

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