© 2017 IJEDR | Volume 5, Issue 4 | ISSN: 2321-9939

Study on Euclidian TSP and asymmetric TSP with reference to graph theory

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Abstract - Like the general TSP, the Euclidean TSP (and therefore the general metric TSP) is NP-complete. However, in some respects it seems to be easier than the general metric TSP. For example, the minimum spanning tree of the graph associated with an instance of the Euclidean TSP is a Euclidean minimum spanning tree, and so can be computed in expected O(n log n) time for n points (considerably less than the number of edges). This enables the simple 2-approximation algorithm for TSP with triangle inequality above to operate more quickly.
In general, for any c > 0, where d is the number of dimensions in the Euclidean space, there is a polynomial-time algorithm that finds a tour of length at most (1 + 1/c) times the optimal for geometric instances of TSP in $O\left(n(\log n)^{(O(c\sqrt{d}))^{d-1}}\right)$ **time; this is called a polynomial-time approximation scheme (PTAS).**

Key words: metric, minimum spanning, dimensions, polynomial-time.

Introduction

In graph theory, the Hadwiger conjecture (or Hadwiger's conjecture) states that, if all proper colorings of an undirected graph G use k or more colors, then one can find k disjoint connected sub graphs of G such that each sub graph is connected by an edge to each other subgraph. Contracting the edges within each of these sub graphs so that each sub graph collapses to a single super vertex produces a complete graph K_k on k vertices as a minor of G.

This conjecture, a far-reaching generalization of the four-color problem, was made by Hugo Hadwiger in 1943 and is still unsolved. Bollobás, Catlin & Erdős (1980) call it "one of the deepest unsolved problems in graph theory."

An equivalent form of the Hadwiger conjecture (the contra positive of the form stated above) is that, if there is no sequence of edge contractions (each merging the two endpoints of some edge into a single super vertex) that brings graph G to the complete graph K_k, then G must have a vertex coloring with $k - 1$ colors.

Note that, in a minimal k-coloring of any graph G, contracting each color class of the coloring to a single vertex will produce a complete graph K_k. However, this contraction process does not produce a minor of G because there is (by definition) no edge between any two vertices in the same color class, thus the contraction is not an edge contraction (which is required for minors). Hadwiger's conjecture states that there exists a different way of properly edge contrasting sets of vertices to single vertices, producing a complete graph K_k, in such a way that all the contracted sets are connected.

If F_k denotes the family of graphs having the property that all minors of graphs in F_k can be $(k - 1)$-colored, then it follows from the Robertson–Seymour theorem that F_k can be characterized by a finite set of forbidden minors. Hadwiger's conjecture is that this set consists of a single forbidden minor, K_k.

The Hadwiger number h (G) of a graph G is the size k of the largest complete graph K_k that is a minor of G (or equivalently can be obtained by contracting edges of G). It is also known as the contraction clique number of G. The Hadwiger conjecture can be stated in simpler algebraic form $\chi(G) \leq h(G)$ where $\chi(G)$ denotes the chromatic number of G.

Special cases and partial results cases where k = 2 is trivial: a graph requires more than one color if and only if it has an edge, and that edge is itself a K_2 minor. The case k = 3 is also easy: the graphs requiring three colors are the non-bipartite graphs, and every non-bipartite graph has an odd cycle, which can be contracted to a 3-cycle, that is, a K_3 minor. In the same paper in which he introduced the conjecture, Hadwiger proved its truth for $k \leq 4$. The graphs with no K_4 minor are the series-parallel graphs and their sub graphs. Each graph of this type has a vertex with at most two incident edges; one can 3-color any such graph by removing one such vertex, coloring the remaining graph recursively, and then adding back and coloring the removed vertex. Because the removed vertex has at most two edges, one of the three colors will always be available to color it when the vertex is added back.

Review of Literature

Graphs are represented graphically by drawing a dot or circle for every vertex, and drawing an arc between two vertices if they are connected by an edge. If the graph is directed, the direction is indicated by drawing an arrow.

A graph drawing should not be confused with the graph itself (the abstract, non-visual structure) as there are several ways to structure the graph drawing. All that matters is which vertices are connected to which others by how many edges and not the exact layout. In practice it is often difficult to decide if two drawings represent the same graph. Depending on the problem domain some layouts may be better suited and easier to understand than others.

The pioneering work of W. T. Tutte was very influential in the subject of graph drawing. Among other achievements, he introduced the use of linear algebraic methods to obtain graph drawings.

Graph drawing also can be said to encompass problems that deal with the crossing number and its various generalizations. The crossing number of a graph is the minimum number of intersections between edges that a drawing of the graph in the plane must contain. For a planar graph, the crossing number is zero by definition.
Drawings on surfaces other than the plane are also studied. There are different ways to store graphs in a computer system. The data structure used depends on both the graph structure and the algorithm used for manipulating the graph. n computer science, a data structure is a particular way of storing and organizing data in a computer so that it can be used efficiently. Different kinds of data structures are suited to different kinds of applications, and some are highly specialized to specific tasks. For example, B-trees are particularly well-suited for implementation of databases, while compiler implementations usually use hash tables to look up identifiers. Ossona de Mendez & Nešetřil (2010) considered that the sparsity/density dichotomy makes it necessary to consider infinite graph classes instead of single graph instances. They defined somewhere a dense graph class has those classes of graphs for which there exists a threshold t such that every complete graph appears as a t-subdivision in a subgraph of a graph in the class. To the opposite, if such a threshold does not exist, the class is nowhere dense. Properties of the nowhere dense vs somewhere dense dichotomy are discussed in Ossona de Mendez & Nešetřil (2012).

Material and Method

The Euclidean TSP, or planar TSP, is the TSP with the distance being the ordinary Euclidean distance. The Euclidean TSP is a particular case of the metric TSP, since distances in a plane obey the triangle inequality. Like the general TSP, the Euclidean TSP (and therefore the general metric TSP) is NP-complete.However, in some respects it seems to be easier than the general metric TSP. For example, the minimum spanning tree of the graph associated with an instance of the Euclidean TSP is a Euclidean minimum spanning tree, and so can be computed in expected O(n log n) time for n points (considerably less than the number of edges). This enables the simple 2-approximation algorithm for TSP with triangle inequality above to operate more quickly. In general, for any c > 0, where d is the number of dimensions in the Euclidean space, there is a polynomial-time algorithm that finds a tour of length at most (1 + 1/c) times the optimal for

Geometric instances of TSP in $O\left(n(\log n)^{(O(c\sqrt{d}))^{d-1}}\right)$ time; this is called a polynomial-time approximation scheme (PTAS). Sanjeev Arora and Joseph S. B. Mitchell were awarded the Gödel Prize in 2010 for their concurrent discovery of PTAS for the Euclidean TSP.
In practice, heuristics with weaker guarantees continue to be used.

Asymmetric TSP

In most cases, the distance between two nodes in the TSP network is the same in both directions. The case where the distance from Ato B is not equal to the distance from B to A is called asymmetric TSP. A practical application of an asymmetric TSP is route optimisation using street-level routing (which is made asymmetric by one-way streets, slip-roads, motorways, etc.).

Solving by conversion to symmetric TSP

Solving an asymmetric TSP graph can be somewhat complex. The following is a 3×3 matrix containing all possible path weights between the nodes A, B and C. One option is to turn an asymmetric matrix of size N into a symmetric matrix of size 2N.
To double the size, each of the nodes in the graph is duplicated, creating a second ghost node. Using duplicate points with very low weights, such as −∞, provides a cheap route "linking" back to the real node and allowing symmetric evaluation to continue. The original 3×3 matrix shown above is visible in the bottom left and the inverse of the original in the top-right. Both copies of the matrix have had their diagonals replaced by the low-cost hop paths, represented by −∞.

	A	B	C	A′	B′	C′
A				−∞	6	5
B				1	−∞	4
C				2	3	−∞

Fig. Symmetric path weights

The original 3×3 matrix would produce two Hamiltonian cycles (a path that visits every node once), namely A-B-C-A [score 9] andA-C-B-A [score 12]. Evaluating the 6×6 symmetric version of the same problem now produces many paths, including A-A′-B-B′-C-C′-A,A-B′-C-A′-A, A-A′-B-C′-A [all score 9 – ∞].
The important thing about each new sequence is that there will be an alternation between dashed (A′,B′,C′) and un-dashed nodes (A, B,C) and that the link to "jump" between any related pair (A-A′) is effectively free. A version of the algorithm could use any weight for theA-A′ path, as long as that weight is lower than all other path weights present in the graph. As the path weight to "jump" must effectively be "free", the value zero (0) could be used to represent this cost—if zero is not being used for another purpose already (such as designating invalid paths). In the two examples above, non-existent paths between nodes are shown as a blank square.

Conclusion

For benchmarking of TSP algorithms, TSPLIB is a library of sample instances of the TSP and related problems is maintained, we can follow the TSPLIB external reference. Many of them are lists of actual cities and layouts of actual printed circuits.

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