A Developed Algorithm For L Fuzzy Multistage Portfolio Optimization Model

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Abstract - Decision makers usually have to face many types of constraints like budget, human resource and etc., while undertaking any new projects. These constraints play a vital role to satisfy their requirements and guarantee profitable growth. The present work mainly indented to assist the decision makers in the task of selecting project portfolios. The workhas been approached by proposing a general nonlinear binary multi-objective mathematical model, which takes into account all the most important factors mentioned in the literature related with Project Portfolio Selection and Scheduling. Due to the existence of uncertainty in different aspects involved in the aforementioned decision task, some fuzzy parameters also incorporated into the model, which allow to represent information not fully known by the decision makers. The resulting problem is both fuzzy and multi objective. The results were compared with graphical tools, which indicated that the proposed model can be effectively incorporated to assist the decision making processes.

Index Terms — fuzzy portfolio- multistage portfolio- optimization model, algorithm- Project portfolio selection and Multiobjective programming fuzzy numbers.

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1. Introduction

Portfolio theory and related topics are among the most investigated fields of research in the economic and financial literatures. In 1952, Markowitz [1] laid the foundation of modern portfolio analysis, The Markowitz's portfolio se-lection theory holds that investors are all pursuing high prospective return, but high returns are inevitably accompanied by high risks. The risk-reducing method is to disperse the fund to assets with different returns and risks. Rational investors should take the lowest risk under the given return rate or seek the largest expected return under the given risk. In real financial markets, there are lots of investors who often invest some assets across multistage, therefore multistage portfolio selection problem often be seen,the problems of multistage portfolio selection have been widely studied [4-6]. But these researches assume that all securities are perfectly divisible and there is no transaction cost.

Conventional portfolio optimization models have an as-assumption that the future condition of stock market can be accurately predicted by historical data. However, no matter how accurate the past data is, this premise will not exist in the financial market due to the high volatility of market environment. To deal quantitatively with imprecise information in making decisions, decision makers are usually provided with information which is characterized by vague linguistic descriptions such as high risk, low profit, high interest rate as proposed by Sheen [2]. With the widely use of fuzzy set theory in [3], people have realized that like to have a decision which not only makes as much profit as possible, but also avoids as much risk as possible. Recently, there has been a continuing effort in designing new approaches for solving the multi-objective programming problem. Abo-Sinna et al. [21] extended the concept of TOPSIS for multiple objective decision making problems to obtain a compromise solution for large-scale multi-objective nonlinear programming problems .

2. Preliminaries

Let us first introduce some basic concepts about fuzzy number, which we need in the following section. The following definition can be found in [18]. A fuzzy number *A* is a fuzzy set of the real line *R* with a normal, fuzzy convex and continuous membership function of bounded support. The family of fuzzy numbers will be denoted by *F*(R), A γ-level set of a fuzzy number *A* is denoted by $[A]^\gamma = \{x \in \mathbb{R}/\mu A \geq \gamma\}$

If $\gamma > 0\gamma > 0$ and $[A]^\gamma =$ cl{ $x \in \mathbb{R}/\mu A \geq \gamma$}(the closure of the support of A) if $\gamma = 0$. It is well known that if *A* is a fuzzy number then$[A]^\gamma$is a compact subset of R for all$\gamma \in [0, 1]$. In the following, we will introduce the notions of possibilistic mean value, variance and covariance of fuzzy numbers introduced in [19] and [7].

Definition .1: Let A $\in F(\mathbb{R})$ be a fuzzy numbers and let $[A]^\gamma = [\, \underline{a}(\gamma), \overline{a}(\gamma)]$ for all $\gamma \in [0, 1]$. The possibilistic mean value of A is defined as E(A) = $\int_0^1 \gamma \, ((\underline{a}(\gamma) + \overline{a}\gamma) \,) d\gamma$ ------(1)

It follows that E(A) is the nearest weighted point to A $\in F(\mathbb{R})$ which is unique.

Proposition.1: Let A,B$\in F(\mathbb{R})$ be two fuzzy numbers and let $\lambda, \mu \in \mathbb{R}$ be real numbers. Then, we have

$E(\lambda A + \mu B) = \lambda E(A) + \mu E(B)$ ----------------------(2)

Definition.2: Let Let A $\in F(\mathbb{R})$ be a fuzzy numbers and let $[A]^\gamma = [\, \underline{a}(\gamma), \overline{a(\gamma)}]$ for all $\gamma \in [0, 1]$.The variance of A is defined as Var (A) = $\int_0^1 \gamma \, ((\underline{a}(\gamma) - E(A))^2 + (\overline{a}(\gamma) - E(A))^2 d\gamma$ ----------------------- (3)

Definition.3: Let Let A ∈ $F(\mathbb{R})$ be a fuzzy numbers and let $[A]^{\gamma} = [\underline{a}(\gamma), \overline{a}\gamma)]$ for all $\gamma \in [0, 1]$.B With $[B]^{\gamma} = [\underline{b}(\gamma), \overline{b}(\gamma)]$ for all $\gamma \in [0, 1]$. The possibilistic covariance of A and B is defined as

Cov (A,B) = $\int_0^1 \gamma \left(\left(\underline{a}(\gamma) - E(A) \right) \left(\underline{b}(\gamma) - E(B) \right) d\gamma + \int_0^1 \gamma \left(\overline{a}(\gamma) - E(A) \right) \left(\overline{b}(\gamma) - E(B) \right) d\gamma \right.$ ----- (4)

According to [7] implication of the following conclusion:

Proposition.2: Let A and B be any two fuzzy number, and let λ, ∈ ℝ , then

Var $(\lambda A + \mu B) = \lambda^2 Var(\emptyset(\lambda)A) + \mu^2 Var(\emptyset(\mu)B) + 2|\lambda\mu| \, Cov(\emptyset(\lambda)A, \emptyset(\mu)B)$---------(5)

$$\text{Where } \emptyset(x) = \begin{cases} -1. \, If x < 0, \\ 0, \; If x = 0 \\ 1. \; If x > 0 \end{cases}$$

Is a sign function of x∈ ℝ.

Furthermore, we can also imply similar conclusion as follows:

Proposition. 3: Let $A_1, A_2, \ldots, A_n$ be n fuzzy numbers, and let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be n real numbers. Then

Var $(\sum_{i=1}^{n} \lambda_i A_i) = \sum_{i=1}^{n} \lambda_i^2 Var(\emptyset(\lambda_i) A_i) + 2 \sum_{i<i=1}^{n} |\lambda_i \mu_i| \, Cov(\emptyset(\lambda_j) A_j)$ ------------------ (6)

$$\text{Where } \emptyset(x) = \begin{cases} -1. \, If x < 0, \\ 0, \; If x = 0 \\ 1. \; If x > 0 \end{cases}$$

is a sign function of $x \in$ R.From the definitions above, we can obtain the following formulas for a trapezoidal fuzzy number $A = (a, b, \alpha, \beta)$with tolerance interval $[a, b]$, left width $\alpha > 0$ and right width $\beta > 0$ if its membership function takes the following form.

$$\mu A(x) = \begin{cases} 1 - \frac{x-a}{\alpha}, & if a - \alpha < x < a; \\ 1 & , if x \in [a, b]; \\ 1 - \frac{x-b}{\beta}, & if b < x < b + \beta; \\ 0, & otherwise \end{cases} \quad \text{------------------- (7)}$$

The γ - level set of A can be computed as follow

$[A]^{\gamma} = [a - (1-\gamma)\alpha, b + (1-\gamma)\beta], \forall \gamma \in [01]$-------------------------------- (8)

By Definition 1 we have the possibilistic mean value of A can be expressed as E(A) = $\frac{b+a}{2} + \frac{\beta-\alpha}{6}$ ----- (9)

From Definition 2, the possibilistic variance of A can be represented

Var (A) = $\left[\frac{b+a}{2} + \frac{\beta-\alpha}{6}\right]^2 + \frac{(\beta+\alpha)^2 + (\beta-\alpha)^2}{72}$ ----------------------- (10)

3. Fuzzy optimization model for multistage portfolio selection problem

In this section, we discuss the possibilistic return, risk of portfolio for the multistage portfolio selection problem.

Assume that there are n risky assets in a financial market for trading, and the returns of assets are denoted as trapezoidal fuzzy numbers. An investor hopes to allocate his/her initial wealth $W1$ among the n risky assets at the beginning of period 1, and obtain the terminal wealth at the end of period T. The investors wealth can be reallocated at every beginning of the following $T - 1$ consecutive time periods. For notational convenience, we first introduce the following notations:

$r_{i,t}$: the return of risky asset i at period t, where $r_{i,t} = (a_{i,t} b_{i,t} \alpha_{i,t} \beta_{i,t})$, the meanings of (a, b, α, β) can refer to preliminaries in section II. $r(t)$: the minimum given return level of the portfolio at period t;

$x_{i,t}$: the crisp form investment proportion of risky asset$i$ at period t;

$x(t)$: the vector of the crisp form portfolio at period t,

where$x(t) = (x_{1,t}, x_{2,t} \ldots x_{n,t})'$;$c_{i,t}$:the unit transaction cost for risky asset i at period t.

$R_{N.t}$:the net return of the portfolio at period t ; w_t: the expected value of the wealth at the beginning of period t ;

$u_{i,t}$: the upper bound constraint of $x_{i,t}, i = 1,2, \ldots . n, t = 1,2, \ldots \;, T$.

3.1 The criteria for multistage portfolio selection problem

In the following subsections, we will introduce the criteria, including return, transaction cost, risk. We will quantify return by the possibilistic mean value, risk by possibilistic variance about the fuzzy return of the asset.

Assume that the whole investment process is self-financing, that is, the investor does not invest the additional capital during the portfolio selection. From the assumption in the previous section,

$r_{i,t} = (a_{i,t} b_{i,t} \alpha_{i,t} \beta_{i,t})(i = 1,2, \ldots . n, t = 1,2, \ldots \;, T)$ are trapezoidal fuzzy numbers. Derived from Definition1, the possibilistic mean value of the portfolio x(t) = $(x_{1,t}, x_{2,t} \ldots x_{n,t})'$ at period t can be expressed as

E $(\sum_{i=1}^{n} x_{i,t} r_{i,t}) = \sum_{i=1}^{n} \left(\frac{b_{i,t} + a_{i,t}}{2} + \frac{\beta_{i,t} - \alpha_{i,t}}{6} \right) x_{i,t}$ t = 1,2, ... , T. --------------- (11)

Let C_t be the transaction cost of the portfolio $x(t) = (x_{1,t}, x_{2,t} \dots x_{n,t})'$ at period t. Then, the possibilistic mean value of the net return of the portfolio $x(t) = (x_{1,t}, x_{2,t} \dots x_{n,t})'$ at period t can be computed as

$E(R_{N,t}) = E(\sum_{i=1}^{n} x_{i,t} r_{i,t} - C_t)$--------------------- (12)

For t = 1,2, . . . , T. From (13) , we have $W_{t+1} = W_1 \prod_{j=1}^{T} E(R_{N,j})$. - - - (13)Thus, the expected value of the terminal wealth at the end of period T by Definition 2 and proposition 2, the cumulative risk of over T period investment can be represented as

$$V(x) = \sum_{t=1}^{T}[\, Var\,(\sum_{i=1}^{n} x_{i,t} r_{i,t})\ ]$$
$$= \sum_{t=1}^{T}[\, \sum_{i=1}^{n} x_{i,t}^2 Var(r_{i,t}) + 2\sum_{i<j=1}^{n} x_{i,t} x_{j,t} cov\,(r_{i,t} r_{j,t})\,]$$
$$= \sum_{t=1}^{T}[\sum_{i=1}^{n} x_{i,t}\left(\frac{b_{i,t}+a_{i,t}}{2} + \frac{\beta_{i,t}-\alpha_{i,t}}{6}\right)]^2 + \sum_{t=1}^{T}[\frac{\sum_{i=1}^{n}[x_{i,t}(\beta_{i,t}+\alpha_{i,t})]^2 + \sum_{i=1}^{n}[x_{i,t}(\beta_{i,t}-\alpha_{i,t})]^2}{72}\ ] \text{ - - - (14)}$$

3.2 The basic formation of multistage portfolio optimization model

Similar to most multistage portfolio optimization models, we first discuss the basic multistage portfolio approach . The basic approach takes the viewpoint of a decision maker that at time t=0 wants to compute and freeze the whole sequence of optimal portfolio x(1), x(2) , . . . x(T). Assume that the transaction cost C_t is a V-shaped function of differences between the t th period portfolio $x(t) = (x_{1,t}, x_{2,t} \dots x_{n,t})'$and the t – 1 th period portfolio x (t-1) = $(x_{1,t-1}, x_{2,t-1}, \dots, x_{n,t-1})'$ .Then , the transaction cost of the portfolio $(x_{1,t}, x_{2,t} \dots x_{n,t})'$ at period t can be expressed as

$C_t = \sum_{i=1}^{n} c_{i,t}|x_{i,t} - x_{i,t-1}|,\ t = 1,2,\dots,T.$ - - - - - - (15)

Thus, we get

$E(R_{N,t}) = \sum_{i=1}^{n}\left(\frac{b_{i,t}+a_{i,t}}{2} + \frac{\beta_{i,t}-\alpha_{i,t}}{6}\right)x_{i,t} - \sum_{i=1}^{n} c_{i,t}|x_{i,t} - x_{i,t-1}|$ --------- (16)

For t = 1,2, . . . , T . Hence , the expected value of the terminal wealth at the end of period T is

$$W_{t+1} = W_1 \prod_{j=1}^{T} E(R_{N,j}) = W_1 \prod_{j=1}^{T}\sum_{i=1}^{n}\left(\frac{b_{i,t}+a_{i,t}}{2} + \frac{\beta_{i,t}-\alpha_{i,t}}{6}\right)x_{i,t} - W_1 \prod_{j=1}^{T}\sum_{i=1}^{n} c_{i,t}|x_{i,t} - x_{i,t-1}|$$

Since, in real world, investors with different preferences may result from different investment strategies. In order to achieve greater flexibility, we propose the following multistage portfolio optimization models, which consist of criteria including return, transaction cost , risk of portfolio.

$max\ W_{t+1} = W_1 \prod_{j=1}^{T}\sum_{i=1}^{n}\left(\frac{b_{i,t}+a_{i,t}}{2} + \frac{\beta_{i,t}-\alpha_{i,t}}{6}\right)x_{i,t} - W_1 \prod_{j=1}^{T}\sum_{i=1}^{n} c_{i,t}|x_{i,t} - x_{i,t-1}|$ - - -(P)

Min V(x) = $\sum_{t=1}^{T}[\sum_{i=1}^{n} x_{i,t}\left(\frac{b_{i,t}+a_{i,t}}{2} + \frac{\beta_{i,t}-\alpha_{i,t}}{6}\right)]^2 + [\sum_{t=1}^{T}\frac{\sum_{i=1}^{n}[x_{i,t}(\beta_{i,t}+\alpha_{i,t})]^2 + \sum_{i=1}^{n}[x_{i,t}(\beta_{i,t}-\alpha_{i,t})]^2}{72}\ ]$

Here s.t.$\sum_{i=1}^{n}\left(\frac{b_{i,t}+a_{i,t}}{2} + \frac{\beta_{i,t}-\alpha_{i,t}}{6}\right)x_{i,t} - \sum_{i=1}^{n} c_{i,t}|x_{i,t} - x_{i,t-1}| \geq r(t)$ - - - (a)

$\sum_{i=1}^{n} x_{i,t} = 1$ - - - (b)

$W_{t+1} = W_t \sum_{i=1}^{n}\left(\frac{b_{i,t}+a_{i,t}}{2} + \frac{\beta_{i,t}-\alpha_{i,t}}{6}\right)x_{i,t} - W_t \sum_{i=1}^{n} c_{i,t}|x_{i,t} - x_{i,t-1}|$ - - - (c)

$0 \leq x_{i,t} \leq u_{i,t}$ i=1,2,... ,T. - - - - (d)

The model consists of two objectives, namely, the maximization of the expected value of the terminal earth and the minimization of the cumulative risk of *T* period investment which is

measured by the sum of the possibilistic variance of the portfolio return at each period. The details of the model (P) are shown in equation (P), where constraint (P)(a) represents the portfolio return must achieve or exceed the given minimum return constraint at each period; constraint(P)(b)indicates the proportion at period *t* sum to one; constraint (P)(c)denotes the wealth accumulation constraint; constraint (P)(d) states the lower and upper bound constraints of$x_{i,t}$. For notational simplicity, we denote the feasible region of the model (P)as$x \in D$.

4. The designed algorithm for our proposed model

In this section, our main idea is presented as follows:

The problem (P) firstly is transformed into the problem (P') using TOPSIS-compromised programming technique, then the corresponding the problem (P') is solved by Genetic algorithm, the specific details of TOPSCP technique and genetic algorithm can be found in the following subsection respectively.

4.1. TOPSIS-compromised technique programming for the proposed model

In this subsection, we will present a novel TOPSIS-compromised programming (TOPSCP) approach for solving the proposed model. To illustrate the advantage of the proposed TOPSIS-compromised programming approach, we will add a comparison analysis with the existing approaches in the next section. First, let us introduce the basic procedure of the designed approach. The fundamental procedure of the designed TOPSIS-compromised programming approach can be summarized as follows:

Step 1: Calculate the ideal and anti-ideal solutions of each objective under the given constraints. For the models (P), we can see that the objective of the cumulative risk of portfolio, V(x), is an attribute of the type of " less is better ", namely, it is a type of cost objective function. Then, its

ideal and anti-ideal solutions can be obtained by solving the following mathematical programming problems, Respectively,

$V^+(x) = min_{x\in D} V(x), V^-(x) = max_{x\in D} V(x)$ - - - (18)

However, the objectives of the expected value of the terminal wealth is the attribute of the type of " more is better ", So the ideal solution of $W^{+}_{T+1} = min_{x\in D} W_{T+1}$ - - - (19)
The anti-ideal solutions of W_{T+1} can be obtained by solving the following mathematical programming problem $W^{-}_{T+1} = max_{x\in D} W_{T+1}$ - - - (20)
Then, the ideal and anti-ideal points of each model can be obtained . Here, we denote $(W^{+}_{T+1}, V^{+}(x))$ as the ideal points of (P), And their anti-ideal points are denoted as $(W^{-}_{T+1}, V^{-}(x))$.
Step2: Use the obtained ideal and anti-ideal solutions to construct normalized positive and negative deviations for each objective, respectively. For the objective of the expected value of the terminal wealth W_{T+1} , its normalized positive and negative deviations are respectively defined as

$d^{+}_{1}(x) = \frac{W_{T+1} - W^{-}_{T+1}}{W^{+}_{T+1}}$ ------------------------ (21) $d^{-}_{1}(x) = \frac{W^{+}_{T+1} - W_{T+1}}{W^{+}_{T+1} - W^{-}_{T+1}}$ --------- (22)

For the objective of the cumulative risk over T period investment V(x), its normalized positive and negative deviations are, respectively, calculated by $d^{+}_{2}(x) = \frac{V^{-}(x) - V(x)}{V^{-}(x)}$ ------ (23) $d^{-}_{2}(x) = \frac{V(x) - V^{+}(x)}{V^{+}(x) - V^{-}(x)}$ ------------- (24)
Step 3: Formulate the Lp-metric and D_p – metric for the objectives of the given problem by aggregating and weighting the normalized positive and negative deviations, respectively. For model (P), the L_p – metric of its objectives is defined as

$$L^{1}_{p} = \{\theta^{p}_{1}[d^{+}_{1}(x)]^{p} + \theta^{p}_{2}[d^{+}_{2}(x)]^{p}\}^{1/p} \text{ --------------(25)}$$

$$D^{1}_{p} = \{\theta^{p}_{1}[d^{-}_{1}(x)]^{p} + \theta^{p}_{2}[d^{-}_{2}(x)]^{p}\}^{1/p} \text{----------------- (26)}$$

Where weights $\theta_1 and \theta_2$ are the relative weights of $W_{T+1}$and V (x) , and p = 1, 2, . . . , ∞ .
Notice that the L_p-metrices above the proposed models measure the weighted sum of the positive deviations from their ideal points, and the D_p – metrices above measure the weighted sum of the negative deviations from their negative points. By varying the value of p, we can obtain different deviation measures for each model such that the greater value assigned to p the more importance given to the biggest individual positive and negative deviations. In other words, both L_p-metric and D_p – metric pfeacj model decrease as the parameter p increases, and greater emphasis is given to the largest positive and negative deviations in forming t he total deviations.
Step 4: Use the simple weighted average method to deal with both L_p-metric and D_p – metric , and then the TOPSIS-compromised problem can be obtained. For model (P), we can obtain its corresponding TOPSIS –compromised problem in the following

$$(P') = \tau\{\theta^{p}_{1}[d^{+}_{1}(x)]^{p} + \theta^{p}_{2}[d^{+}_{2}(x)]^{p}\}^{1/p} - (1-\tau)\{\theta^{p}_{1}[d^{-}_{1}(x)]^{p} + \theta^{p}_{2}[d^{-}_{2}(x)]^{p}\}^{1/p}$$

$$\text{s.t.}x\in D, \theta_1 + = 1\ \theta_j \in [0,1], j = 1,2. \text{ ------ (27)}$$

where$\tau \in (0,1)$ can be considered as the prefence coefficient of L_p-metric . The greater τ is, the more preference for L_p-metric the investor is.

4.2 Genetic algorithm design for our model

Notice that the model (P') is a complex nonlinear programming problem, if using the traditional algorithm may fail to obtain the optimal solutions. In order to avoid getting stuck at a local optimal solution, we will design a genetic algorithm to solve them. In this section, we design a genetic algorithm with penalty term to solve our model. Without loss of generality, let us take the model (P) for example. Here, we first troduce its representation, initialization, evaluation function, selection, crossover and mutation.

Representation and initialization. In this algorithm, we encode a randomly generated solution x= ($x_{1,1}$; . . .; $x_{n,1}$; . . .; . . ., $x_{n,T}$) into chromosome by real-valued representation C = ($c_{1,1}$, . . . , $c_{n,1}$; $c_{1,T}$, . . . , $c_{n,T}$), where the genes $c_{i,t}$ (I = 1,2, . . . n; t = 1, 2, . . . , T) are restricted in the corresponding variable bounds. Repeat this operation pop-size times, then pop-size chromosome, C_1, C_2, . . . ,$C_{pop\text{-}size}$, can be obtain***ed.***
Evaluation function with penaly term. Since the randomly generated pop-size solutions may not satisfy the feasible region. Inspired by [28], we construct the following evaluation function with penalty term

$$\text{Eval (x) = exp (}\omega\text{ f(x) – K p(x)) ---------------- (28)}$$

Where ω is a positive constrant, K is a sufficiently large positive number, f(x) denotes the value of the objective function p(x) represents a penalty term, and it is defined

p(x) = { 0 , $if x \in feasibleregion, \sum_{t=1}^{T} \max(g_t(x), 0)^2 + \sum_{t=1}^{T} \max(|h_t(x)|, 0)^2\ otherwise.$ ----------- (29)

here $g_t(x) = \sum_{i=1}^{n}\left(\frac{b_{i,t}+a_{i,t}}{2} + \frac{\beta_{i,t}-\alpha_{i,t}}{6}\right)x_{i,t} - \sum_{i=1}^{n} c_{i,t}|x_{i,t} - x_{i,t-1}| - r(t)$,
$h_t(x) = \sum_{i=1}^{n} x_{i,t} - 1$ denote the inequality and equality constraints of (P), respectively.
*Selection process.*The selection process is based on the proportional selection. In order to prevent the populations from degenerating during iteration, both parents and their immediate offsprings are candidates for the new generation. We compare the parents with their offsprings by evaluating their fitness values to select the fitter ones to store in a mating pool. After the comparison operation, we can obtain
pop-size chromosomes with higher fitness values. Then, we perform the proportional selection operation. The chromosome k is selected for a new population by the following probability$P_s(x_k) = \frac{eva;(x_k)}{\sum_{k=1}^{pop-size} eval(x_k)}$ ------ (30)

In this process, the chromosomes of the current population with higher fitness values have higher chance as the parents to reproduce the offsprings. Crossover operation. The crossover process is based on arithmetic crossover. We first denote the crossover probability of the genetic algorithm as *Pc* . The crossover operation performs as follows. Generate a random number *U* from interval (0, 1) and the chromosome $C\ k(k = 1, 2, \cdots, pop-size)$ is selected as a parent provided that $v < P_c$. Repeat this process pop-size times and

$Pc\ pop-size$ chromosomes are expected to be selected to perform the crossover operation. The crossover operation on *C*1 and *C*2 will generate two offspring's $C_1'\ and\ C_2'$ as follows: $C_1' = vC_1 + (1 - v)C_2$ ----- (31)

$$C_2' = (1 - v)C_1 + vC_2 \quad \text{----------- (32)}$$

Conclusion

This paper proposes multistage portfolio model in fuzzy environment. For solving the proposed model, a improved algorithm is designed and numerical example is given to illustrate the effectiveness of the proposed algorithm. The results of numerical example indicate that the proposed algorithm is effective. Though this algorithm only provide an approximated result, it is a good method to solve larger and more complex problems

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